From Wisdom to Data

– Philosophical Atlas on Visual Representations of Knowledge

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Coleção Transversal

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Contents

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Preface
From Data to Wisdom and back

This book collects results from the research project "From Data to Wisdom. Philosophizing Data Visualizations in the Middle Ages and Early Modernity" funded by the FCT (Fundação para a Ciência e a Tecnologia), POCI-01-0145-FEDER-029717.[1] The project had a double purpose: (1) to create a repository of medieval visualizations of information and knowledge, proposing a distinction between different kinds of representation: relational schemes, knowledge-experience simulations, data (storage/indices/tables/charts), elemental schemes, text-diagrams, and demonstrative graphics; and (2) to make these visualizations interact with modern and contemporary visualizations, in particular contemporary data visualizations. More generally, the aim of the project was to show how the history of Western thought is not only a history of texts but also (and perhaps increasingly) a history of images and visual representations of concepts and knowledge.

The purpose of these texts is to collect in a single volume the multiplicity of approaches, perspectives and contexts in which the research project has been developed. To do this, we believe, a synthesis is not necessary. A totalizing synthesis would be contrary to the very spirit and results of the project itself. Indeed, one of the theoretical results achieved is the idea that contemporary visualizations have abandoned the desire for absolute, synthesizing representation in favor of a representation that is always imperfect and, so to speak, "in the making." The very idea that data visualizations are representative, i.e., that they refer to "reality", has been widely discussed and criticized as well. Our intention, therefore, is to present a series of analogies between texts and images. In short, this book will be a *mise en abyme* of the even incomplete synthesis, but for this very reason richer in meaning, which in our opinion is, or at least should be, at the heart of today's visual representations of knowledge. Our theoretical model is not that of the summa, but rather that of Aby Warburg's *Mnemosyne Atlas*. Warburg's atlas was entirely made of images put next to one anoother not according to a single culture, discipline, or timespan, but according to affinities related to cultural memory *tout court*. Similarly, our intent is not to offer an orderly account of the data visualization history, but rather to disrupt it, to "reclaim the power of the atlas as a fragmentary, unfinished site of resistance" (Vickers & Allado-McDowell 2020, p.20).

1 We would like to thank the whole team of the project "From Data to Wisdom" (to access the complete list of researchers involved, please consult: https://ifilosofia.up.pt/proj/fdtw/steering_committee).

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Interviews

Centrum meum ubique locorum: Diagrams as Depicting Actions, an Interview with Jeffrey F. Hamburger

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1. In the text accompanying the penultimate full-page miniature in the Rothschild Canticles (fol. 104r) – **Image 1**, you identified a maxim that had a long tradition in medieval philosophy: *Centrum meum ubique, circumferentia nusquam*. Despite the intricate construction of this manuscript, one could think the first two images representing the Liberal Arts (ff. 14v-15r) have little do to with the contemplative and theological aspects of this mystical miscellany; do you think this is the case? Even more, could the Liberal Arts show the way to shape thought and devotion visually in the Middle Ages?

The secret to understanding the complex and seemingly very diverse iconographic program of the Rothschild Canticles, illuminated ca. 1300, lies, first, in recognizing that its images trace the full trajectory of mystical ascent, from the propaedeutics or foundation represented by the study of the Seven Liberal Arts, the quadrivium and trivium alike, all the way to the summit of mystical experience, marked by not one, but as many as twenty individual representations of the Trinity, an extraordinary number when one considers the difficulty of picturing the paradoxical combination of three persons in one. The key to understanding the Trinitarian series, in turn, lies in seeing its dazzling and ever-changing combination of anthropomorphic and non-anthropomorphic imagery, the latter consisting of inventive combinations of suns, flames, light rays, clouds, and veils, as a playing out, first, of the process of perichoresis (περιχώρησις *perikhōrēsis*, “rotation”;

Latin: *circumincessio*) performed by the three persons, Father, Son, and Holy Ghost, and, second and simultaneously, of the concomitant process by which the soul, in this case identified with the reader and viewer of the manuscript, is herself taken up into and subsumed by the Trinity so as to experience (not simply understand) for herself this ultimately incomprehensible and invisible mystery. The miniatures provide a simulacrum of mystical ascent, employing metaphorical imagery derived from the Bible, biblical exegesis, poetry, and theological commentaries and mystical tracts. The text on folio 103v, in effect a cento or chain of snippets from various texts rather than a description of the adjoining miniature or a source text in any straightforward sense, combines the traditional maxim that "God is like a circle whose center is everywhere and whose circumference nowhere" – popularized by Alain of Lille and derived, perhaps, from the *Liber XXIV philosophorum*, a short Latin treatise of anonymous authorship consisting of two dozen definitions of what God is, accompanied by commentary – with three other excerpts. Of these, the first is a paraphrase of an antiphon (suggesting that it was recalled from memory and addressed to a monastic reader) for the feast of the Dedication of the Church, "Gloriosum et terrible nomen tuum," appropriate in this context in that it celebrates the sublime unknowability of the Godhead; the second, a somewhat garbled prose rendition of an epigram, which in the original reads "Quod Deus est, scimus. Quid sit, si scire velimus,/ Contra nos imus. Qui cum sit summus et imus,/ Ultimus et primus, satis est; plus scire nequimus; "We know that God is; if we wish to know what he is,/ We go against ourselves. That he is the highest and the lowest,/ The last and the first, is enough; we can know no more").[1] As noted by Barbara Newman, this text found its way into the *Carmina* of Petrus Pictor, a canon of Saint-Omer in the diocese of Thérouanne, the very region to which the *Rothschild Canticles* can, on the basis of stylistic comparisons, be localized. For the third and final text, which in translation reads "God always was and shall be without end; where he always was, there he is now, and where he is now, there he was then," no source has yet been identified. Redolent of the liturgical formula expressing eternity, "in saecula seculorum," "unto the ages of ages," it could conceivably have been invented by the compiler. All of these texts lend expression to the paradoxical concepts of the divinity's infinite expansion in space as well as time, as well as to its inaccessibility to human comprehension. None, however, prescribe what appears on the facing page: a Trinitarian implosion, as I have previously described it, in which a sunburst akin to those found throughout the manuscript – including, it should be noted, on the first folio representing the Arts turns in on itself, as if in a supernova. Whereas the practitioners of Grammar, Arithmetic, and Geometry remain very much focused on their earthbound activities, the astronomer at upper right, who aspires to higher things, looks up to the sun and moon, which appear within a bank of clouds above the towers and crenellations of the architectural frame. It is as if the motif of the sun provides the alpha and omega of the entire iconographic program. It is, however, not natural light that the contemplative perceives. In the full-page miniature, an inverted sunburst, golden beams of light stream inward to converge on the center only to be met by the corresponding rays that extend from that principal point. At the top, left, and right of the circumference of the surrounding circle, three smaller sunbursts symbolize the three persons in the individuality, while triplets of intertwined flames between them point to their threefold unity. Father and Son, represented as human beings, their hands grasping the circle, their feet dangling in the void, are joined by the barely visible dove of the Holy Spirit at the summit of the circle; all three are shown in the midst of a divine disappearing act: an astonishing enactment of the concept of the *Deus absconditus*. The small clouds set into the corner of the miniature indicate that this passage from visibility to invisibility takes place not in any space governed by Cartesian coordinates but rather in a place beyond all knowing, but one that is, simultaneously, opened up, as if in a Baroque *sotto in sù* ceiling, to the contemplative gaze, a process enacted in corporeal terms by the twisting, dancing figure of the seer in the smaller miniature on the facing page. This gesticulating figure, derived from a representation of the Annunciation to the Shepherds, provides an exemplar of response for the reader, who thereby is encouraged to turn from reading (the *verso*) to seeing (the *recto*) in a simulacrum of contemplative ascent.

2. One aspect of medieval diagrams that you have encountered concerns the relation between schemata and pictorial narrative. Would you mind expanding further on the connection between medieval graphical speculation and storytelling in relation to the *carmen figuratum* added in the early thirteenth century at the front of a twelfth-century copy of the Etymologies of Isidore of Seville (Douai, Bibliothèque municipale, ms. 318, f. 1r) – **Image 2**?

The intermingling of text and image is fundamental to the aesthetic of medieval images, whether in historiated initials or in the form of pictures integrating inscriptions. In this manuscript from Marchiennes in northern France, written in the third quarter of the twelfth century, which, in addition to Isidore's *Etymologies,* contains a number of poems (the picture poem added, to judge from the scripts employed, in the first half of the thirteenth century), constitutes an appropriate supplement and preface in that, like Isidore's work, it too makes play with words. It does so, however, in ways that are pictorial as well as poetic. Like a diagram, the poem, which is dedicated to the Virgin, combines visual and verbal elements in the form of a grid or lattice employed as a multiplier of meaning. In adopting this form, the verses (unpublished as far as I can determine) employ a traditional format for poetry of this sort, whose origins can be traced to Antiquity (and in particular to the court poet to Constantine, Publibius Optantianus Porphyrius). In the Middle Ages, the most famous (and ambitious) exemplar of the genre was *In honorem sanctae crucis* (In honor of the Holy Cross) by the Carolingian monk, Hrabanus Maurus, of which copies were produced right down to the date of the earliest printed edition in the sixteenth century. The example from Marchiennes pays tribute to this famous model and its late antique antecedents by adopting and expanding the format of the eighteenth of Porphyrius' poem (*Carmina* 18). There is, however, one critical difference: whereas Porphyrius both superimposes a square on his poem (thereby matching the typical format of a late-antique codex) and simultaneously uses it to frame and subdivide his verses into four smaller quadrants, each divided in turn by diagonals, making for a total of sixteen small compartments set against and within the fabric of the larger poem, of which the *versus intexti* are of a piece, the later poem, in keeping with the vertical rectangular format characteristic of most medieval codices, doubles this format by superimposing two such squares, each divided into four compartments, making a total of eight interlocking spatial and poetic units, but without the underlying poetic fabric. While more complex in its geometry in that the smaller squares are divided not only by diagonals, but also by crosses, so that each one itself has eight compartments, and the poem as a whole sixty-four, in another fundamental respect the medieval imitation abandons the greater degree of difficulty constituted by the demands of the model, in which letters and words have to perform multiple functions, depending on their position. There is also, not surprisingly, a basic divergence in terms of subject matter. Instead of honoring the emperor Augustus, as does Porphyrius, in verses all of which openning with words beginning with the same letter as his name (*Alme, Aurea, Aucta, Aurea, Aonios, Augusti, Aonii*), the later work is written in praise of the Virgin and employs repeated words that begin (or end) with the letters M and O, which alternate with one another if the verses are read horizontally or diagonally, but which appear in alternating uniform columns running top to bottom. As in the model, the readings offered by the structure are multivalent. Rather than reading from top to bottom, left to right, the reader can proceed horizontally, vertically or diagonally from any of the intersections marked by the capital letters, written in red, both of which serve as either the first or last letter (and sometimes both simultaneously) of the words that converge on them. Surrounding the entire structure are additional verses that apparently identify the name of their author, Walter, and adopt a penitential tone. These somewhat inelegant framing verses begin at the upper left "Lector Walteri deposce deum misereri" (Reader, pray that God have mercy on Walter) and continue "Qui postquam cineri dederit sua membram foveri" (Who, after [his death], will give his limbs to be warmed as ashes), before concluding this first section "Amota pena requie mereatur aniena" (Peace would require that pain be removed). The corresponding verses running along the outer margin read "Christum si quis amat, sicut sacra littera clamat./ Mundum in hoc mundo,/ Stet sicut corpore mundo./ Atque fide sana de restans cuncta prophana." The opening of this section, "If any man love Christ," represents a paraphrase of I Corinthians 16:22 (If any man love not our Lord Jesus Christ); the continuation proclaims its source: "as holy scripture proclaims." The formula "littera clamat" recalls a passage from Alain of Lille's *Anticlaudianus* (III.522–525) in which the poet describes a picture of which the letter paradoxically proclaims "the artificers who enclose within set limits the expanse, measurements, weights and boundaries of things, and, with a like process of reason, examine the air, sky, stars, seas, lands."[2] If Walter in fact knew this passage, he could well have conceived of himself as an poetic artificer who not only lent a speaking voice to a visual form, but who also, akin to the geometer Thales praised by Alain, sought to capture the mysteries of the Virgin Mary within the limits and boundaries defined by the geometric matrix he imposed on himself. The remainder of the inscription running along this side of the page reads: "Adorned in this world, let him stand like a corpse in the world and in sound faith abstaining from all profane things." The sentiments expressed could not be more different from those of the distant model. If Porphyrius exalts the emperor, whose triumphs will be praised to the skies (*Alme tuas laurus aetas sustollet in astra*) and whose light suffices as a reward for the poet (*Aurea lux vatum, silvae mihi praemia serva*), Walter humbly offers up his verses to the Virgin, all while spurning earthly vanities. Indeed, the difficulty that the structure of his adopted format enforced could itself be construed as a form of penitential exercise.

3. The hybrid nature of diagrams that mix word with image, thereby giving expression to the combination of the visible and invisible (whether concepts or theological mysteries), constituted a challenge but also an opportunity for medieval artists. Given your extensive experience in dealing with such issues, what is the relationship between medieval diagrams, tables and charts and such didactic devices as the monastic game of the "Battle of Numbers" (Rithmomachia) – **Image 3**?

The diagrammatic character of the game board employed for the game of *Rithmomachia*, whose purpose was to offer monks an alternative to supposedly sinful games such as dice and whose rules were designed to inculcate Boethian mathematical principles as laid out in his *De institutione arithmetica*, offers an opportunity to consider the ludic character of diagrams. In the case of a board game designed in the name of devout didacticism, the play in question is literal. It also, however, acquires an aesthetic dimension akin to that identified by Reviel Netz in the style of calculation (and of writing about such calculation) in Hellenistic mathematics, in which, in his words, there is a "fascination ... with creating a rich texture of obscure and seemingly pointless numerical calculation," as well as "with such themes that go beyond the boundaries of geometry, either connecting it to other scientific genres or indeed connecting it to non-scientific genres such as poetry."[3] Substitute "theology" or "exegesis" for "poetry," and one has a good description of the kind of imaginative, even paradoxical, geometry often found in medieval treatises on such topics as the Trinity, whether by Joachim of Fiore, Thierry of Chartres, or Nicholas of Cusa, for whom geometry as metaphor and geometry as proof often converged in the creation of dazzling diagrams. A practice such as that represented by *Rithmomachia* would both have inculcated and encouraged the tendency to think in such a manner. According to the surviving treatises that outline the rules of the game, whose origins can be traced to the eleventh century, the game gave access to Wisdom in the form of the rules of number that governed God's creation. Also known as the Philosopher's Game, its rules were first recorded by the monk "Asilo," who can probably be identified as Adelbero (c. 1010–1090), bishop of Würzburg, and Hermannus Contractus (1013–1054) of Reichenau. A diagram of the game board, including some of the ratios on which the movement and capture of pieces are based, follows a short exposition of the game by Odo of Tournai (1060–1113) in a mid-twelfth century miscellany of scientific texts from Normandy Avranches, Bibliothèque municipale, ms. 235, f. 77v), perhaps from the monastery of Mont-Saint-Michel or that at Coutances. But, as can be seen from the example reproduced here, images illustrating the game continued to be printed in the early modern period. Played on a board eight squares wide and sixteen long, the game pitted odd versus even numbers, a topic central to Boethius's treatise and demonstrated by the most elaborate of its explanatory diagrams, in which numerical relationships based on multiplication are visualized within rows and columns defined in terms of "latitude" and "longitude." In the game, the pieces took the form of rounds, triangles, squares, and pyramids (the latter composite pieces made by stacking several others atop one another), in short, the forms of the Boethian solids. One could win in a variety of ways: by achieving certain "harmonies" of pieces corresponding to three types of numerical proportion or progression, arithmetic, geometric, and harmonic. Each move had to conform to a specific set of mathematical operations. Just as Netz examines to what extent the protocols of mathematical calculation and proof in the Hellenistic world harmonized with aesthetic principles found in other arenas, such as poetry, so too, one can ask to what extent the habits inculcated by practices such as mathematically-based board games might have informed the creation of medieval images. The role of geometry in shaping medieval art represents something of a locus classicus in discussions of medieval artistry, especially in the Romanesque, although frames (especially architectural frames) also provide one of the defining features of Gothic art. Following Henri Focillon, Jurgis Baltrušaitis insisted on what he called a "law of the frame" that governed and constrained figural representation, whether of individual figures or of larger narrative configurations. Meyer Schapiro famously took exception to this supposed principal, arguing for a far more dynamic understanding of the relationship between figure and frame in which opposing forces of tension and compression worked to lend life to the image. In the end, the dynamism of diagrammatic procedures tends to support Schapiro's position. As even a superficial glance at the pages within Villard de Honnecourt's so-called sketchbook demonstrate, geometrical schematism also played an indisputable role in the very process by which medieval artists not only produced but also remembered how to produce a wide range of images. Moreover, in manuscripts, the same ruling patterns that structured the overall mise-en-page of the text often also provided a scaffolding, if not a constraint, for the accompanying illumination. Ornament provides another arena in which an underlying geometry, often not perceptible to the naked eye, lend stylized representations of botanical motifs an uncanny combination of stability and vitality. At issue is not whether such armatures or matrices constrained the imagination of medieval artists, but rather to what extent they enabled it. Geometry, far from fixed, was fundamentally generative in character.

4. In your research, you have devoted significant efforts to determining the audience for books of devotion, prayer, and contemplation that are full of visual narratives. Are medieval nuns and courtly women the main target of this visual teaching?

The issue of the audience for medieval narrative imagery is enormously complex. In principle, Gregory's letter to Serenus (*Registrum*, Bk. XI, letter 105) – a prescription so widely circulated that it came to be known as the *Gregorium dictum* – governed such questions. In it, the pope instructed the bishop of Marseille that "pictorial representation is made use of in churches for this reason; that such as are ignorant of letters may at least read by looking at the walls what they cannot read in books."[4] Walking a fine line between, on the one hand, the adoration of a pictorial image, which he forbade, and, on the other, iconoclasm, which he also prohibited, Gregory argued that narrative imagery in ecclesiastical spaces, which in the sixth century would, on the model of the great churches in Rome, above all else have taken the form of programs of mural painting principally in the nave, served as an appropriate instrument of edification for an audience stratified in part by their access (or lack thereof) to the written word. Even in so far as one is dealing with church decoration of Gregory's day, one has to take his prescription with a grain of salt. To begin with, his letter, no less than later references to it, has to be understood in the context of contemporary art, theology, and politics. As numerous studies have shown, medieval images were capable of making arguments in their own right equivalent to a form of visual exegesis. Chief among these was the form of parallelism manifested within typological programs that in systematic fashion paired events from the Old Testament with those in the New, thereby arguing that the former was a foreshadowing of the latter. The disposition of figures both within and among images also carried significance as did the refined application of such formal means as color, frontality, size, and hierarchy. In short, monumental wall painting represented nothing more than a straightforward illustration of scripture (or hagiography); it spoke in its own voice. In the case of manuscript illumination, such questions potentially become still more complicated in so far as the very same codices contain texts to which, presumably, the readers of the book also had access. In such cases, the issue of literacy does not apply. With some significant exceptions (for example, Anglo-Saxon illumination of vernacular texts in the eleventh century), one has, beginning in the twelfth century, to distinguish between manuscripts written in Latin and in the various vernaculars. In some cases, of which the richly illuminated St. Albans Psalter is perhaps the most famous and certainly among the very earliest examples, the very same manuscript contains both Latin (in this case, the Psalms and canticles) and the vernacular (the Old French life of St. Alexis, the earliest extant written text in French, which in the manuscript is described as a song (*cancun*)), thereby at the very least rooting its transmission in oral performance. Making the matter still more complicated is the fact that the manuscript's codicology indicates that it almost certainly was not originally made for its presumed recipient, the anchoress, Christina of Markyate, for whom the quire containing the Life of St. Alexis, which displays meaningful parallels to the life story of Christina herself, was, it seems, inserted in a second stage of production. Despite such qualifications, one can nonetheless observe that, beginning in the twelfth century, many devotional books, mainly if not exclusively psalters, containing sequences of serial narrative imagery were made for either aristocratic women or for nuns who were members of burgeoning female orders. Monastic rules and treatises of this period explicitly distinguish between the needs of nuns and novices as opposed to full-fledged monks when it comes to the use of imagery as a prop for devotion. Here too, one must take such prescriptions with a grain of salt; not only do they reflect deep-seated gender biases, themselves rooted in ingrained preconceptions about male and female physiology and intellectual capacity, they also, as in the case of early medieval mural painting, are contradicted by the subtlety and complexity of the pictorial programs, which prove themselves perfectly capable of fashioning sophisticated theological arguments. Similar tendencies can also be observed as far as early Books of Hours, the successor to the psalter in terms of its popularity as the devotional book of choice, are concerned: many, if hardly all, of the earliest examples (including the *Cursus Sanctae Mariae*, New York, Pierpont Morgan Library and Museum, MS M.739, very likely made for St. Hedwig of Silesia) were made for aristocratic women who either emulated or adopted a monastic routine, often as widows. In this particular instance, the various prayer texts, all in Latin, are prefaced by a lengthy cycle of narrative imagery spanning the whole of salvation history in which the images are paired with captions in the German vernacular, suggesting a link between vernacularization and visualization. In this context, perhaps the key point is that when it came to such books, but also other forms of devotional imagery, among them miracle-working icons and so-called *Andachtsbilder*, the affective piety of monastic women, often if not exclusively within the context of the *cura monialium* (pastoral care of nuns), led the way in defining what would later become normative practices among the laity at large. They did so, not because women were by nature more emotional or bound to sensory experience, but rather because in many contexts, such practices represented what was expected of women by cultural convention (there are not powerful exceptions). In the end, however, by which in this context is meant the later Middle Ages, women often had the last word, or, in this case, one could say, image: the oft-noted flood of images that marks late medieval piety and that, in turn, provoked a violent reaction from Protestant reformers, was in large part prepared by women's innovative approach to and use of visual imagery, perhaps most amply documented in the prescriptive as well as descriptive and poetic texts produced in the realm of female mystical piety, in many of which colorful visions play a central role.

5. Inspired in part by the work of Otto Pächt, you have pointed out the "iconic turn" regarding medieval mystical representations and logical oppositions. Farther afield, you also refer to Charles S. Peirce's semiotic theory. Are diagrams the fundamental framework of relational thinking?

Your question raises many additional ones, among them: need we, when we speak of the "iconic turn," focus on figural imagery or do various forms of abstraction, medieval or modern, also fall under the same rubric? What do mystical and visionary imagery (not the same thing, also in terms of medieval theories of vision) have in common with diagrammatic images, of which logic diagrams were among the most important in medieval pedagogy and practice? What can modern theories of the diagram, whether in logic, physics, or computer and cognitive science, to name but a few of the many arenas in which diagrams are widely deployed, teach us about medieval diagrams (and vice versa)? I am often astonished by the degree to which modern students of diagrams in such fields blithely assume that the use of diagrams remains more or less concomitant with the scientific revolution of the early modern period, which in turn is linked to the use of printing to disseminate knowledge with a previously unprecedented ease and efficiency. If only such scholars and scientists would look back farther to consider the history and application of diagrams in the Middle Ages and Antiquity! If they did so, they would find not only evidence of their pervasive application across a wide array of disciplines, in effect, the entire medieval curriculum, but also of their integration into profound considerations of such closely related topics as cognition and the visualization of knowledge. The works of Nicholas of Cusa, who often is characterized as a forerunner of modernity but whose diagrammatic habits remained deeply rooted in the tradition of medieval Neo-Platonism right back to

Late Antiquity, offer among the most compelling examples (a topic I touch on in my recent book, *Color in Cusanus*. Hiersemann, 2021). To bring together the discourses on medieval and modern diagrams was the primary motivation behind another recent publication, *Diagramming Devotion: Berthold of Nuremberg's transformation of Hrabanus Maurus's poems in praise of the Cross* (University of Chicago Press, 2020). Among the diagrams that Berthold, a German Dominican of the late thirteenth century, took as his models in explaining and elaborating the *carmina figurata* of Hrabanus were logic diagrams such as the universally known square of opposition, a staple of school learning throughout the Middle Ages, and the so-called Tree of Porphyry. He employed such models not only because they provided matrices for image-making that lay readily to hand – models with which he could presume his primary audience of fellow Dominicans would be familiar – but also, still more importantly, because they provided a convenient vehicle for his own creative process. Although the two topics might strike us as strange bedfellows, in Berthold's hands, logic and meditation merged. To try to answer my own questions, prompted by yours, how can the abstractions represented by and through diagrams be related to the iconic turn? To the extent that one of the principal thrusts of this turn has been to locate agency in images, diagrams offer a good example in so far as they are capable not simply of recording but also of generating knowledge in the first place. Aristotle (*De memoria et reminiscentia* 449b31–450a1) already recognized the constitutive function of diagrams in cognition; indeed, he declared that it was impossible to think without images (by which he meant diagrams). As to what diagrams might have in common with mystical habits of visualization, one way of answering the question would be to turn to images such as those representing the Trinity in the *Rothschild Canticles* (the origin of my own interest in the subject), in which geometric armatures play a critical role in the principal artist's astonishing attempts to lend visual form to concepts and objects that ultimately lie beyond the scope of visualization. The etiolated, extended forms of diagrams, in which thin lines border on invisibility, proved ideally suited to the representation of the divine as well as to its interface with the limited capacities of the human intellect, not to mention the senses. In like fashion, even if twisted into oxymoronic forms, geometrical "proofs" of theological principles lend themselves to supposedly incontrovertible demonstrations of proof. If the tension between faith and understanding provided one of the main arenas in which theological debate took place, diagrams provided one possible point at which these two guiding principles could meet. As for Peirce, anyone who has read his work will have come away with the impression that not only was he enamored of Aristotle, whom he, incidentally, claimed to understand better than anyone else, but also that his own writing often took on many attributes of medieval Scholasticism, not least in his obsessive application of threefold patterns within his semiotic system, which extended the familiar taxonomy of icon, index, and symbol to as many as sixty-six subcategories of sign. Indeed, Peirce converted from Unitarianism to Trinitarianism. With reference to the iconic turn, Peirce's thinking (and his drawings) have been invoked, specifically by Horst Bredekamp in relation to his theory of the "Picture Act" (parallel to the concept of speech acts) as "the very essence of picture act theory."[5] Bredekamp cites Peirce's observation that "a great distinguishing feature of the icon is that by the direct observation of it other truths concerning its object can be discovered than those which suffice to determine its construction" (Charles S. Peirce, Manuscripts on Existential Graphs, Houghton Library, Harvard University, MS Am 1632 [787]). As I have noted elsewhere, Bredekamp employs this statement to reinforce his own claims regarding the character of all images and to defend himself against critiques of his theory of the *Bildakt* as representing a form of animism.[6] Bredekamp argues that "This kind of critique is grounded neither in essential philosophical traditions nor evolutionary theory. Its vain but understandable vehemence stems from a painful inability to overcome a constructivist conceptualization of subjectivity and its world-defining telos" (p. 5). Here, however, it is important to take into account the context from which Bredekamp's quotation is taken, namely, the immediately preceding statement by Peirce that "particularly deserving of notice are icons in which the likeness is aided by conventional rules. Thus, an algebraic formula is an icon, rendered such by the rules of commutation, association, and distribution of the symbols. It may seem at first glance that it is an arbitrary classification to call an algebraic expression an icon; that it might as well, or better, be regarded as a compound conventional sign. But it is not so." Peirce acknowledges, rather than rejects, the contributing role of conventions. In his characteristically Aristotelian manner, he also makes clear that he is speaking of algebraic symbols (and, by extension, diagrams), not pictures per se. Furthermore, he spells out that his focus is on the reliability of reason, not on what Bredekamp defines as the "energeia" of an object or picture "that approaches human beings independently, and ultimately liberates them from the ego-logics of constructivism, representationalism, and neurocentrism, and the rivalry between image and language as well as that between the visual and the tactile" (p 32).[7] To your last question, namely, "are diagrams the fundamental framework of relational thinking," one could answer with both Aristotle and Peirce that all thinking, relational and otherwise, is fundamentally diagrammatic.

Notes

1 For the identification of this text, which eluded me in my publication of 1990, see Newman, B. (2013). Contemplating the Trinity: Text, Image, and the Origins of the Rothschild Canticles. *Gesta*, 52/2, 133–159, esp. 146.

2 Translation from Alanus de Insulis, *Anticlaudianus*; or The Good and Perfect Man, trans. with commentary by Sheridan, J. (1973). *Mediaeval Sources in Translation* 14 (Pontifical Institute of Mediaeval Studies, p. 115. Sheridan's translation of the opening of these verses ("There the painter's mark proclaims ...") fails to capture the literal sense, according to which it is the letter of the picture that speaks: "Illic artifices pictoris littera clamat ..." For the arts of painting and poetry in Alain's work and in this passage in particular, see in Simpson, J. (1995). *Sciences and the Self in Medieval Poetry: Alain of Lille's Anticlaudianus and John Gower's Confessio amantis*. Cambridge University Press, p. 241. The phrase applied in a similar fashion recurs elsewhere, e.g., in the poem *De quatuor evangelistarum proprietatibus et significationibus* by Petrus Riga; see Beichner, P. (1949). The Cursor Mundi and Petrus Riga. *Speculum*, 24/2, 239–250, esp. 242: "Effigiat Marcum leo, cuius littera clamat/ Quanta surrexit ui tua, Criste, caro." In this case, the letters that speak might be construed as a titulus. Indeed, the verses are employed as such for the figure of Mark on the cross of the Abbey of Clairmarais in Saint-Omer, for which see Favreau, R. (1991). Le thème iconographique du lion dans les inscriptions médiévales. *Comptes rendus des séances de l'Académie des Inscriptions et Belles-Lettres*, 135/3, 613–636, esp. 631.

3 Netz, R. (2009). L*udic Proof: Greek Mathematics and the Alexandrian Aesthetic*. Cambridge University Press, p. xi.

4 Translation from https://www.newadvent.org/fathers/360209105.htm (accessed 28.7.2021).

5 Bredekamp, H. (2014). The Picture Act: Tradition, Horizon, Philosophy. In S. Marienberg, & J. Trabant (Eds.), *Bildakt at the Warburg Institute*, Actus et Imago 12. pp. 3–32, De Gruyter., esp. p. 14.

6 What follows is taken more or less verbatim from Hamburger, J. (2019). *Diagramming Devotion*, pp. 221 and 335–336.

7 On the passage in question, see also Paolucci, C. (2017). Semiotics, Schemata, Diagrams, and Graphs: A New Form of Diagrammatic Kantism by Peirce. In K. Hull & R. Atkins (Eds.), *Peirce on Perception and Reasoning: From Icons to Logic*, Routledge Studies in American Philosophy 10 (pp. 74–85). Routlegde, and Gerner, A. (2017). Diagrammatic Thinking. *Atlas of Transformation*, accessed on July 12th, 2017: http://monumenttotransformation.org/atlas-of-transformation/html/d/diagramma tic-thinking/diagrammatic-thinking-alexander-gerner.html: "Diagrammatic thinking therefore first makes the principle of diagrammatic reasoning accessible by introducing new elements through iconicity and through the operations of 'hypostatic abstraction' and theoretical abstraction. One has to be careful not to adopt a trivial similarity definition when speaking about diagrams as part of iconicity. From a non-trivial standpoint similarity cannot be equated with 'identity' with the object, nor can similarity be psychologized to refer to merely subjective judgments or feelings of resemblance."

Ars Memorativa as the Genesis of Information Design: A Conversation with Manuel Lima

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Introduction

Manuel Lima is one of the most prominent figures of data visualization since the publication of *Visual Complexity* (Lima, 2011). In this conversation, Manuel Lima traces back the origin of data visualization to *Ars Memorativa*, an ancient mnemonic technique to organize information and facilitate its recall. Going back to the origins is an obsession that brought him to collect and arrange into books images of information design from both physical and digital archives. By doing this, Manuel Lima tackled issues related to digital objects and their creation, use, and preservation, with a point of view capable of combining the passion for visualizing information and the profession of UX designer.

This conversation, which took place between Lisbon and Milan on Wednesday, July 28th, 2021, comes from a blurb that Manuel Lima wrote for *Mapping Affinities* (Rodighiero, 2021). The discussion is part of the project From Data to Wisdom, and is supported by Fundação para a Ciência e a Tecnologia through the grant POCI-01-0145-FEDER-029717 and the Swiss National Science Foundation through the grant 194442.

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1. Dear Manuel, I would like to start the interview with a very simple question: how did it all start? When did you fall in love with data visualization?

My love of data visualization started during my Master's Degree of Fine Arts at Parsons School of Design in New York, it was a two-year program. During that period, a teacher of mine gave a lecture on information architecture. He presented this diagram called the Understanding Spectrum (Shedroff, [1994] 1999), which basically shows how data leads into information, information into knowledge, and knowledge ultimately into wisdom – **Image 4**. Even though my background is actually in industrial design and product design – I'm familiar with Politecnico di Milano, which is a great school for industrial design, amongst other fields – so in building physical things, I was just super interested in being part of the process of bridging information and knowledge. That's the key challenge, arguably one of the most critical challenges of the century. Then, I immediately started planning different projects, already thinking about data visualization in an attempt to make sense and visualize systems.

I remember doing this project on visualizing both a computer virus and a human virus, which is now an urgent topic for all of us due to COVID-19. I got really into epidemiology and the dreaded diseases. I was fascinated by how the spread of a computer virus is actually similar to a human virus, contaminating hubs like within a network. I remember I was reading Albert Barabasi's book *Linked* (Barabási, 2002), and Steven Johnson's *Emergence* (Johnson, 2001). This slowly got me into networks, and I became obsessed with these structures.

And for my thesis, I was trying to build up this visualization tool to understand how information spreads within blogs. This was 15 years ago and blogs were very recent. Blogs were just coming out and everyone was saying, "It's the end of the media as we know it." But I was really more interested in how blogs themselves became a social laboratory for research, which is something that social scientists have been trying to do for ages. Today we have this perfect lab to understand how information spreads from point A to point B, which people are 'contaminated' by this information, and what's the overall pattern of dissemination.

When I was doing this research for my thesis, I started collecting examples of network visualization, which intrigued me because they underlie the structure of the blogosphere and the world. My obsessive-compulsive nature was pushing me to collect examples not just from that specific domain, but also from everything: from biological networks, computer systems, and social networks. That was really the genesis. Once I graduated, I felt the body of research was too good to keep it to myself, and that's how the book started. As I was collecting some of those examples on visual complexity, I realized this would be a great book because there are several examples that are actually, unfortunately, disappearing. And the book was serving multiple purposes – I think I published *Visual Complexity* (Lima, 2011) five years after the website – but one reason was really to preserve digital maps for future generations because many of them are disappearing. Visual Complexity is a way of preserving some of that content.

Then, my curiosity about the origin of things prompted me to write my two other books. I was always trying to see who was the first person to think about network diagrams or create a network diagram, and that led me to investigate tree diagrams, which are the topic of my second book, the *Book of Trees* (Lima, 2014). The same mental process of going back to the origins, in trying to understand the very first diagrams that humans created, inspired me to write about the circle (Lima, 2017).

2. This is truly beautiful. By looking for the origin of data visualization you moved from contemporary to ancient visuals. Is there a specific reason that led you to investigate the past, especially the Middle Ages?

I think that in the *Book of Trees* and the *Book of Circles* there's an implicit attempt to demystify the idea that we are at the clinical stage of civilization, and data visualization is a brand-new thing that has emerged in the last hundred years as if nothing has happened before. I hope my two books displace this misconception because it's simply not true. We have been doing this for centuries, if not millennia. Now we just have better tools and more data, but the challenge is still the same: to make the invisible visible and to find even patterns in information.

The very sense of being overwhelmed with information, which is such a common thing today, it's also not completely new. There's a book titled *Too Much to Know* (Blair, 2010) that I absolutely love because it details the anxiety that medieval people were facing with this avalanche of information coming from ancient Rome and Greece. There was all this incredible explosion of content, and people had to make sense of it. Then, there were also the developments in printing that increased book production. There is this sense that we were creating so much knowledge, so much information that it was impossible for any single human to digest. There was a similar type of anxiety as we have today with big data, that there's too much information out there.

3. I would ask you to develop this point. If this overwhelming information already existed in the past, it means that information design is a pretty old practice. Did you finally find its origins?

I always go back to what is known as *Ars Memorativa* – **Image 5**, which is also called the Art of Memory in English. *Ars Memorativa* is a set of mnemonic techniques used by schoolers back then – most of them religious because that was the nature of knowledge – to memorize information and recall it at later times. And I do think that Ars Memorativa is really the genesis of information design.

Since Ancient Greece, the great thinkers of the past have defined a lot of critical design principles that, even today, are still used and embraced by interaction design, UX design, and information design. Ideas like the chunks or hierarchy were defined during the Ars Memorativa movement.

It was during this time, when this avalanche of information was coming from ancient Rome and Greece, that some people realized that visual communication and diagrammatic communication were notable ways to deal with this data. There were outliers, a few people that wanted to have all the information in a diagrammatic form because it was just easier to consume. They were intentional about their goals: diagrams were meant not to accompany the text but to replace it altogether. So I think that was really the birth of *Ars Memorativa* that shaped information design as we know it today, and this was roughly nine centuries ago.

4. You told us that you studied industrial design and UX design, but information design was totally new when you started to explore it. I know that a lot of designers like you and I did this transition easily. So one of my questions is: what is the relation between user experience design and information design?

I run into this question all the time. Even looking back, I think my professional life was divided in two parallel tracks. On the one hand, I was doing research, lectures, and teaching related to data visualization and information design. On the other hand, my 9:00–5:00 job was working for big companies like Google and Microsoft on interaction design and UX design. Though they were parallel tracks, I have always felt that they were pretty connected. User experience design and data visualization are interconnected through interaction. One thing that really differentiates modern-day information design is the requirement of interactive design. Otherwise, we are falling into our traditional graphic design on paper. Once you embrace interaction, once you're creating digital experiences that allow you to interact, to filter, to drill down different aspects of a visualization, inevitably, you enter the domain of interaction design and user experience, which is all about understanding the flow, usability, accessibility, and things of that nature. There's a huge overlap. Interaction is really the glue between user experience design and data visualization.

5. I'm sure that you know the work of the philosopher Raymond Llull for many reasons, including your Iberian origin. Something that struck me about Llull, in addition to his capacity to excel in diagrammatic communication, was his initiative to send books all around Europe to get visibility. Manuel, I have the impression that your books reached a lot of readers. Did being an author change your life?

It's always hard to pinpoint the causality of certain things in your life. Of course, my books definitely led to more lectures, more invitations for events, and speaking arrangements. I think that the TED talk was through *The Book of Trees*. One of the recruiters for TED saw the book and invited me to give a lecture.

One inevitable effect was more speaking engagements, for sure. First of all, it's almost a compulsion to just put it out. I'm living with this thing, and I have to put it out. I cannot live with myself if I just don't put this thing out into the world. There is this sense that other writers have also conveyed, the same sense of a need. But for me, the second thing is really touching people. What happens when you write a book like that is that you touch other people without even getting to know them. That's really the main reason why I do it. I love when I get an email or a message, or a LinkedIn post of someone saying, "I read this book and it changed my life," or "I changed careers because of this book." It's fun. Due to the nature of the work that I end up doing in places like Google, Microsoft, it's really ephemeral. I like the digital environment. It's always changing. If I look back, there's really nothing I can show my kids. Whatever software I created has been replaced, it's already long gone. The books are at least one way of lasting a little bit longer, of keeping something that will hopefully outlast myself, and that future generations can consume it.

6. It's the second time you touched on this idea of preserving digital artifacts. I totally agree with you when you say that today printed material lasts longer than digital. Librarians have the same perspective on it. What are your concerns about the life of digital objects?

It's a huge problem, Dario. It's really something that more of us should be worried about if we care about culture and preservation for the future. Many of our current digital creations are just vanishing as we speak.

When I was doing research for *Visual Complexity*, it was easier to find a medieval illustration created eight centuries ago than a digital execution created 10 or 20 years ago. The reasons are really multifaceted. Sometimes the plugin doesn't work anymore; Adobe Flash doesn't exist anymore, it was abandoned. You have to assume that the authors or the companies still exist, and they are diligently keeping the image or the code somewhere. It's not on a server anymore, it's probably on a local drive, who knows? These digital objects just keep on vanishing. They keep on deteriorating, and we are losing them. And not to mention, there are few cases where you even still have images, but the resolution is so tiny. Do you remember how images back then were? They were so tiny that they could not even be included in books.

There's this notion that the digital will save us. It's really dangerous because we don't realize how ephemeral the digital really is – how unsafe it is. It's prone to every type of attack. We are seeing this all over the world today. Digital is also no guarantee of preservation. It's likely there's less of a guarantee of preservation than paper or stone. When I go to the British Museum, I see incredible examples of preservation in cuneiform texts from Babylonian tablets dating 5,000 years old – **Image 6**. Some papers don't even last that long. Do you really think today that we can look back in eight centuries and see anything done today using our digital tools? I have strong doubts about that.

7. I totally agree with that. My Master thesis project, which is relatively recent, was created with Flash and I can't open it anymore. I preserved the code and some screenshots in my cloud storage. Tell me about your new book, and I think this may be a good ending for this conversation. Can you talk a little bit about this project?

Yeah, I can. It's going to be different. My visual phase has ended. This book likely will not have a single image. It's going to be more focused on design; it's more about the responsibility of design in this new world. The book is going to cover ethics, environmental responsibility, social responsibility, and things like that. It's going to be around nine chapters, and each chapter is meant to demystify a preconceived idea that we might have about design.

8. Is this connected to your recent change of life? I know that you moved back to your home country after having spent a considerable amount of time abroad.

It is. I spent the last 10 years in New York and the last 20 years outside of Portugal, where I'm from originally, and it's been quite a journey. We have two small daughters, Chloe and Caniba, and it was at some point in the midst of COVID-19. It was crazy to see New York City just closed. We were locked in a two-bedroom apartment with two children screaming and the schools were closed. It was just too much, and I realized it was time to go. We came to Lisbon in June last year, and we had an amazing summer. Portugal back then was completely open, it was almost like a life as normal. That was really, really good.

I have mixed feelings. New York City will always be a city that we love. And now we actually have dual citizenship. So I'm Portuguese-American. I'm sure it's always going to be part of our lives. But so far, Lisbon is really fun: the weather, the food, the beaches, it's hard to beat. I'm expecting a visit soon, Dario.

Sure, with great pleasure. Yes, I totally understand your choice, and it was the same for me. Coming back is sort of beautiful, even if things have changed. I look forward to your book, and hope there will be time to meet you in person. It has been a pleasure.

Imagination, Images, and Imaginaries: a Dialogue with Jos De Mul

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Interviewee:
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Imagination

1. The first part of this dialogue is focused on the notion of imagination. It is from your work (to which I would add the work of Paul Ricoeur) that I have learned that (1) imagination is not *creatio ex nihilo*, but rather recombination and (2) imagination is always technologically (and digitally) externalized. I am thinking, for instance, of your 2009 book chapter "The Work of Art in the Age of Digital Recombination". Could you tell me a little bit more about your understanding of imagination, its relationship with technology, and the importance you attribute to authors like Kant, Dilthey, and Cassirer in your research?

I have been writing a new book on database for some time now. The reason why the book is still not finished is that it completely went out of hand in a way when I rediscovered Cassirer. I wrote an additional chapter on Cassirer and it's about 90 pages, so it's almost a complete book on Cassirer now. But it was important for me because all things fell in that place. The first chapter of my book is called "The Medialization of Experience." And I start with Kant in the book.

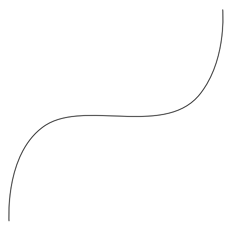
And what is the medium for Kant? The medium is the human reason. However, there is in Kant a presupposition that all people have the same apriori forms. This idea became a problem very soon after, because of the worldview's historicization in the 19th century. The second person I discuss after Kant is Dilthey. The first thing Dilthey says is that these apriori are historically and culturally variable. We live in different worlds, literally: not in the sense that we look differently to the same world, but our phenomenological worlds are truly different. Furthermore, according to Dilthey, Kant is focusing too much on theoretical reasoning. But knowledge is not only natural scientific knowledge. We also have cultural knowledge. Human and social sciences (*Geisteswissenschaften*) apprehend the world in a different way than the natural sciences: human experience is a nexus of knowing, willing and feeling, and in each faculty, the bodily component is very important. Moreover, what Kant does not consider is that our thinking is not empty reasoning. It is always taking place in a natural language, and natural language is an external medium that influences the way we experience the world.

In sum, we could identify three elements in Dilthey's criticism to Kant: first, the historicization of the apriori; second, what I call "vitalization" of the apriori – experiences are bodily experiences; third, externalization of the apriori. In my book, I show how in the 19th and 20th centuries, these three themes are becoming main themes, especially in continental philosophy. Hence, media philosophy is a kind of *philosophia prima* of the 20th century.

Cassirer further develops Dilthey's threefold transformation of transcendental philosophy. Whereas Kant's transcendental schemata of the imagination are fixed a priori of scientific reason, for Cassirer there are multiple "symbolic forms." Each of these symbolic forms imagine time, space, and categories like causality in fundamentally different ways.

Cassirer started his career as a neo-Kantian, and the neo-Kantians were very focused, epistemologically, on natural sciences. But after he became a member of the *Kulturwissenschaftliche Bibliothek Warburg*, Cassirer started to focus on other symbolizations of the world, like art, religion, mythology, technology, etc. These forms are historical and cultural in nature. However, already in Cassirer's early, still neo-Kantian work on Einstein, we find the germs of his later philosophy of symbolic forms. Kant started from a universal apriori of space and time based on the mathematics of the Greek; Einstein, on the other hand, used new kinds of non-Euclidean mathematics, so it apparently contradicted Kant. But Cassirer made an interesting move. Kant was not wrong, he says. Well, maybe he was wrong in the sense that that he did not see the historical dimensions of our a priori. But Einstein supports Kant insofar as he showed that the way we experience the world depends on our apriori forms – only these forms develop in time, and his theory of relativity was based on newly developed non-Euclidean geometries.

Cassirer especially became interesting for my own project when I discovered his philosophy of technology. I never realized before that he had a philosophy of technology. Technology is not only the subject of Cassirer's somewhat neglected article "Form and Technology" (1930), but also plays a prominent role in the second volume of *The Philosophy of Symbolic Forms* (1925). The book is about mythology and there, Cassirer argues, mythology is the first form of technology. Mythology tries to control the world like technology, only it's still "wishful thinking". Myths try to control the world by story. Myths represent an important step because a prehistoric hammer, for example, is a kind of transitional object from mythology to technology because hammers were seen as instruments of the Gods, possessing and providing magical power to its users. But he says this was also the first notion of technology. A hammer is something you use *to do* something, and that's almost already the hammer analysis of Heidegger in *Being and Time*. I don't think this is coincidental. Heidegger wrote a positive review of this book of Cassirer, emphasizing the practical dimension of it. Human reason is not sheer theoretical thinking, but it has to do with practice.



What is a symbolic form for Cassirer? It is a way to symbolize the world, "a seeing as..." The same figure (such as the line on the left) can be interpreted as a magical symbol, an aesthetic expression, or the visualization of a mathematical function. Art, religion, language, law, and science belong to the most fundamental ways of symbolizing the world. Cassirer claims that these symbolic forms are universal, and universal means two things for him. They are universal in the way you can apply them to everything, so everything can become subject of language, religion, science, etc. But they are also universal, according to him, because you find them in all cultures. They are universal, but at the same time, they have many different manifestations. Every culture has a language. Every culture has a religion. Every culture has some sort of technology and science. But their manifestation is historically and culturally variable.

What I especially found very fascinating reading Cassirer is that, according to him, there is a kind of evolution of symbolic forms: everything starts, in a way, with mythical thinking. He says that's a kind of first moment in time that human beings start to symbolize the world. They are no longer just like animals reacting to external stimuli, rather, they are trying to grasp the world in a mental way. I already mentioned the hammer, which has a kind of ritualistic aspect. But the hammer is also a starting point for technology, and for science, as its use also brings along a notion of causality. Religion can also be understood as an evolution of the mythical thinking when it gradually starts with personifications of the powers of nature up to the notion of a god. For Cassirer, we still find this mythical element in modern thinking, for example, in technology. Technology still has a magical element in it. We hope to control the world with the help of technology; once we had magic hammers, now we have magic computers and magic algorithms, so to say.

In my book *Cyberspace Odyssey* (2002), I have also analyzed this magical aspect of technology: every advanced form of technology is like magic. The telephone is a kind of magical thing in two senses. First, because it's effective. I mean, it works. But on the other hand, it is magic because it is mystery. You don't really know how it works, it's magic in that sense.

Although Cassirer argues that we can discern a pattern in the development of symbolic forms towards an increasing abstraction, progress is not guaranteed. In his last book, *The Myth of State* (1946), Cassirer says: if you look at Nazi Germany, you see that the idea of the political was taken over by a mythical thinking. Think of the mythology of the thousand-year empire, the magical figure of the leader, etc. Politics, which was originally a kind of emancipation from mythological thinking, and developed into an autonomous realm, so to say, falls back into magical thinking. And also technology, he says, was used by the Nazis in a very clever way: radio and TV and film in order to spread their mythical ideology. And so, these different symbolic forms, they have one origin, but they can always again intermingle. That fits in very nicely with what you said in your discussion with Lemmens (see Romele's article "The Transcendental of Technology is Said in Many Ways", 2021). Indeed, the technological is a transcendental form, but there are many other transcendentals of technology. I introduce a term for it in my book, which is in the spirit of Cassirer, although he doesn't use the term himself, to summarize his position. I call it "transcendental perspectivism."

2. Thank you Jos. And what about recombination? I mean, isn't the very historicized, externalized, and embodied nature of human imagination also the reason for its recombinatory nature? Aren't we always, precisely for these three dimensions (historicity, vitality, and technicity) "standing on the shoulders of giants" even when it comes to our most authentic creations? Don't you think that digital machines, that seem more and more be able to observe regular patterns in our creative works, and seem increasingly able to imitate our most creative gestures, are demonstrating this? Should we be afraid of it?

In my opinion, there is no reason for fearing the recombinatory nature of our imagination. The *Ars (Re)Combinatoria* consists of all possible (re)combinations, of which the vast majority does not exist (yet). Take the alphabet: only 24 letters, in that sense every past, future and possible utterance is 'just' a recombination of these 24 letters. Does this limit imagination? Think of Borges "Library of Babel." dealing with a library consisting of all possible books of 410 pages, 40 lines, each having 80 characters, that is, 1,312,000 characters, written with the 25 characters of the Spanish language (22 letter, blank, dot, comma). The number of books then is 25 to the power 1,312,000 ($2^{51,312,000}$). If we consider that according to cosmologists the universe consists of 10 to the power 80 (10^{80}) atoms, this number is negligible compared to the number of books in the library of Babel!

There is also a "Library of Mendel" (Dennett), which combines all possible genetic combinations. Interesting point: according to Dennett not all combinations are logically possible (existing and not-existing at the same point), not all logically possible combinations are physically possible, not all physically possible combinations are chemically possible, not all chemically possible combinations are biologically possible (a flying horse: herbivores do not take in enough energy to fly), not all biologically possible combinations are historically possible (path dependency: think of descendants of extinct species like dinosaurs or the dodo).

3. This is an interesting point, because Dennet's perspective seems to be more realistic. It considers not only the possibilities in principle, but also the "affordances." In principle, many planets in our galaxy could host some form of life. But if you look at the affordances, the possibilities are less and less, but everything becomes also much more realistic. Maybe my question here is: if we look at our use of database and algorithms, especially for scientific discovery and technological innovation today, could we say that we are leaving the "Borges age" and entering the "Dennet age"? In other words, are we more realistic than ten or twenty years ago about the possibilities and the limits? It's a bit like the infinite monkey theorem, which is the claim that a monkey typing random keys on a typewriter keyboard for an infinite amount of time will almost certainly type any given text, such as the complete works of William Shakespeare. You certainly know that someone has run the experiment and it did not work. In a month, the monkeys produced nothing but five total pages largely consisting of the letter "S". Moreover, the lead male began hitting the keyboard with a stone, and other monkeys followed by smearing it. What I mean is that Dennet does not deny the recombinatory principle, but he makes it more real and less giddy than Borges'.

Indeed, I think we are in the very first stage of the Dennet's age. Take, for example, genetic engineering, which is putting this idea of (re)combination into practice. I also think about mass customization: now you order a specific car and then they make it with the engine you like the most, the radio system and the color etc. you wish.

Cassirer says symbolization in a way comes down to the use of metaphor. And of course, the use of metaphor is imagination. We are back to imagination. So, metaphor is a kind of imagination in which we symbolize the world. If we say "the heart is a motor" we design a new worldview in a way. We know how important metaphors are. Well, these are conceptual metaphors. We use them to grasp the world. The idea Catherine Hayles introduced is that there are also *material* metaphors that do something in the world. If we replace a defective heart with a mechanical equivalent, the conceptual metaphor – the mechanistic worldview – becomes a material one. My former Ph.D. student Marianne van den Boomen has applied this idea to computers in her thesis *Transcoding the Digital* (2014). For her, computers are material metaphors because you can do something in the world, for example, if you take a computer program that is used by a biologist, geneticists for manipulation of genes. For example, if we call a CRISPR-Cas9 a programmable scissors, we have a conceptual metaphor. But by connecting your computer to a DNA synthesizer, it becomes a material metaphor. You can change reality with it. I would say that's the moment that you enter the Dennet age. In the case of Borges, it's still a very interesting play of imagination and artwork – **Image 7.**

I want to add one more thing, which has to do with monkeys, evolution, and self-organization. If you look at this, these monkeys out there, the idea is that if they have enough time, they can write the complete works of William Shakespeare. Well, you can also be a little bit more modest and say, how long will it take before they will write one single sonnet of Shakespeare? Then it will still take many billions of years before the first sonnet of Shakespeare will appear, that is: much longer than the ca. 13 billion years our universe exists. So the idea that completely at random machines would produce something like a sonnet is neglectable. Well, that's hardly possible. That is, I think, also the message of Borges' story because the library contains all possible books. Also, the book with the ultimate truth of the universe. But you will never find it because the number of books in the library is literally hyper-astronomical, bigger than the number of atoms in the universe. So there must be another principle as well. And that is self-organization.

Richard Dawkins defends a very mechanistical notion of evolution: for him, it is just random mutations. And then he also mentions this example of the monkeys and he says we can speed it up by, for example, you do the type randomly and as soon as one of the letters of the sonnet is in the right place, you fixate it. You will have the complete sonnet in a couple of weeks instead of billions of years. This is a principle of self-organization.

And there is a self-organization also in "'biological databases", also known as organisms. From an evolutionary perspective, biological databases are not fixed, but adaptive systems that change because of their interaction with the environment. But it also applies to cultural databases like a natural language. Think of the database of language which in the age of globalization adapts itself to the multicultural environment (in the Dutch language, for example, many new words entered from the Indonesian and Surinam languages (during the colonial period), Turkish and Moroccan (because of the multiculturalization), and above all English (because of the global domination of the Anglo-Saxon culture)). This opens new paths in the database, novelties in the vocabulary, grammar, and style.

I think the novelty often emerges when two different symbolic forms merge. You can use technology in many different cultural form contexts, so to say, and so you can use the computer to produce art, you can use it as an economical instrument, you can use it as a scientific tool. And the same applies to AI algorithms, of course.

Images

4. The second part of this dialogue is devoted to images, and in particular diagrams. For me, the link between imagination and images, diagrams in particular, is obvious: diagrams are externalizations of knowledge that do not merely represent knowledge, but "operate" it as well. In this sense, they are imaginative – where imagination is mainly understood as *"ars recombinatoria."* If we consider medieval diagrams, are we then dealing with proto-imaginative machines? After all, what is being done today is in some way a continuation by other means of what Ramon Llull and many others were already attempting to do in the 13th century. Another point: before this dialogue, I have asked you to choose one of the images from our catalogue. You chose this one (see below). Why?

Yes, I chose this image – **Image 8** – and then I read Michael Stolberg's *Uroscopy in Early Modern Europe* (2015). According to him, urine functioned as data (dead matter, extracted from body), which made it possible to predict and control. Objective, visual evidence instead of subjective feelings! Uroscopy offered visual evidence of long-held beliefs about the body. It was widely presumed, for instance, that illness could result from morbid matter collected and settled within the body. Cloudy urine was thought to provide evidence of this phenomenon. The practice only began to lose favor altogether by the 19th century, and Stolberg calls this decline the result of "uroscepsis." Interesting parallel with data: Stolberg states that nowadays it seems absurd to think that we can predict and control solely on the basis of the color of urine. Nowadays we magically believe that we can predict and control solely on the basis of data (another case of dead matter extracted from the body: objective, visual evidence).

5. What I like about diagrams, is that they can be observed or "read" according to different levels of abstraction. There is, for example, a primary level, which I would say is almost chromatic. For instance, your image of the urine hit me, first of all, because of the color. Second, there is a symbolic level in which I recognize forms and in case I can attribute a meaning to them. Finally, there is a diagrammatical level in which I "use" the image as an instrument, a sort of combinatorial and recombinantorial machine, so to say.

I think again of Cassirer in the sense that he says it is possible to have one visual Gestalt – like the line I've shown you before – and interpret it in many different ways so connect it with different meanings. Maybe this is both the weakness and the richness of the image: you can always have new interpretations of the image.

6. On the one hand, you can interpret it in different ways. I think it depends on the context. But on the other hand, images like texts have their legitimate and illegitimate interpretations. In the case of diagrams, I can make them work as diagrams,

as (re)combinational machines. This is not the case of other images. I cannot take a painting by van Gogh and use it as a diagram. Diagrams I can use them both as paintings, let's say, and as diagrams.

Indeed, a diagram is something between an image and a mental idea. What is also interesting in Cassirer for me is that he demonstrates that science starts very concretely. There are concrete objects, and you try to do something with them. You have a metal, you want to melt it, or you have a tree, and you want to make a house of it, you try to understand and deal with concrete objects. Originally, mathematics was also used to do that. For example, geometry was often used for measuring land. So, it was originally used for very concrete purposes. According to Cassirer, the greatness of Leibniz is that he saw mathematics as no longer related to the concrete world, but to possible worlds. And in a way, diagrams are about possible worlds. They do not give one specific image of the world, but different possibilities. Diagrams are not representations of the reality, but simulations of it, not showing what is, but what is possible. Normally, images for Cassirer are the lowest part in knowledge because they just represent reality. Diagrams are already an attempt to do things more abstract, but still in the realm of the image. So yeah, they are already transcending the image in a more spiritual way, in a more conceptual way, but they do it still in the language of the image. And that's why I see them as a kind of transitory phenomenon. Mathematics, the language of modern science, is the most abstract type of symbolization.

7. Technically, the image of the urine is not even a diagram. It is more like a map, a cartography of the different diseases and the different relations between diseases and urine colors.

Yeah, but it is already database-like: you have two different elements, they are all ordered in a certain way and the colors are from light to dark, so to say. There is an organization. When you ask what a database is, I think there are three elements in it. First, a *collection of elements*. Second, a *model of organization* of those elements (e.g., a hierarchical, network or relational database model). Third, a *database management system*, a concrete application of a database, with specific roles and rights to insert, browse, change and delete data. A classical telephone book is already a database: you have different elements (all the names of the telephone users), and it has a model of ordering (alphabetical order). It is a system, in the sense that you can use it to look for the telephone number of a person, but it is also not very flexible. A little bit like the urine boxes because you cannot recombine them in many ways. A second way to do it is to make an index card box with telephone numbers. It is more flexible because you can order it in different ways (by name, postal code, number), insert or delete cards etc. But it is still time-consuming to re-order such a database. The third step is the database in the computer program because if you have an Excel sheet, you can change it with one click. You can change the database. You can order by name, street address, city, postal code, etc. And so, you say that this diagram or map is a first step. Like the telephone book is a primitive step on the way to the flexible database. The most flexible form is the relational database, which enables you to combine and recombine all atomized data elements. Borges' Library of Babel is actually such a relational database (you can find several versions of it on the web). The relational database is in a way the concretization of the ideal of knowledge of Leibniz.

Imaginaries

8. I would now like to go to the last part of our dialogue, which is devoted to imaginaries. In a 1999 article, you introduced the notion of "Informatization of the worldview." This informatization represented for you an alternative to the previous mechanistic worldview. Do you think that something has changed in the last 20 years? I was wondering if the way we deal with meaning today is quite different. Meaning is today a matter of emergence from chaos, while the chaos is still present and often represented. Meaning is a matter of probability. In the informational age, meaning was much more "linear" I would say. Chaos was feared and eliminated. Today, we show it; so, meaning is always partial in comparison to the chaos that is around. A couple of years ago I wrote an article about this, titled "The Datafication of the Worldview" (2020), which was a kind of integration in this direction to your previous reflections on the topic.

I see your point, but I think basically the origin of informatics was the same. Look at Shannon. Shannon was an engineer. For him, the problem was, for example, when you want to transfer a message to a telephone line or another telegraph line, that there is a lot of noise in the transfer of the data, so there is a lot of chaos. And what he wanted to do was to create order in the chaos. This was quite simple in that time, you could say, because the chaos was in one line, the telephone line. Today, we have a lot of data, and we look for interesting patterns – and maybe, like Shannon we don't know what we want with it, but we are just interested in the efficient syntactical patterns.

9. Still, I would say that today we have a higher tolerance for chaos. I mean, look at Shannon. The point was to eliminate noise for him. He wanted a message to arrive as clear as possible from point A to point B. Today we accept a certain degree of chaos, and we accept it as part of this world.

Of course, we are dealing today with a statistical notion of truth. But this notion was already introduced by quantum mechanics in the beginning of the 20th century. Einstein could not accept it: he famously said that God does not play dice with the universe. So, I am wondering whether we really are accepting chaos so much more. I think we are forced in a way because we are forced to accept that the world is not that organized as we would like to have it. And that's a kind of mythical wish that the world is organized. During the Human Genome Project, it turned out that only about 2 percent of the DNA has coding capacities. And the other 98 percent, they called it junk DNA. This 98 percent is maybe remainders from our evolutionary past. Waste that is still in the cell, but which has no use anymore. Well, later they discovered it was not chaos at all because big parts of it have regulatory functions. And so it was a big relief. Fortunately, it's not chaos!

The Master Algorithm (the idea introduced by Pedro Domingo in his 2015 book) will always remain a fantasy. But of course, fantasies can help you. So, they have a function. An imaginary function!

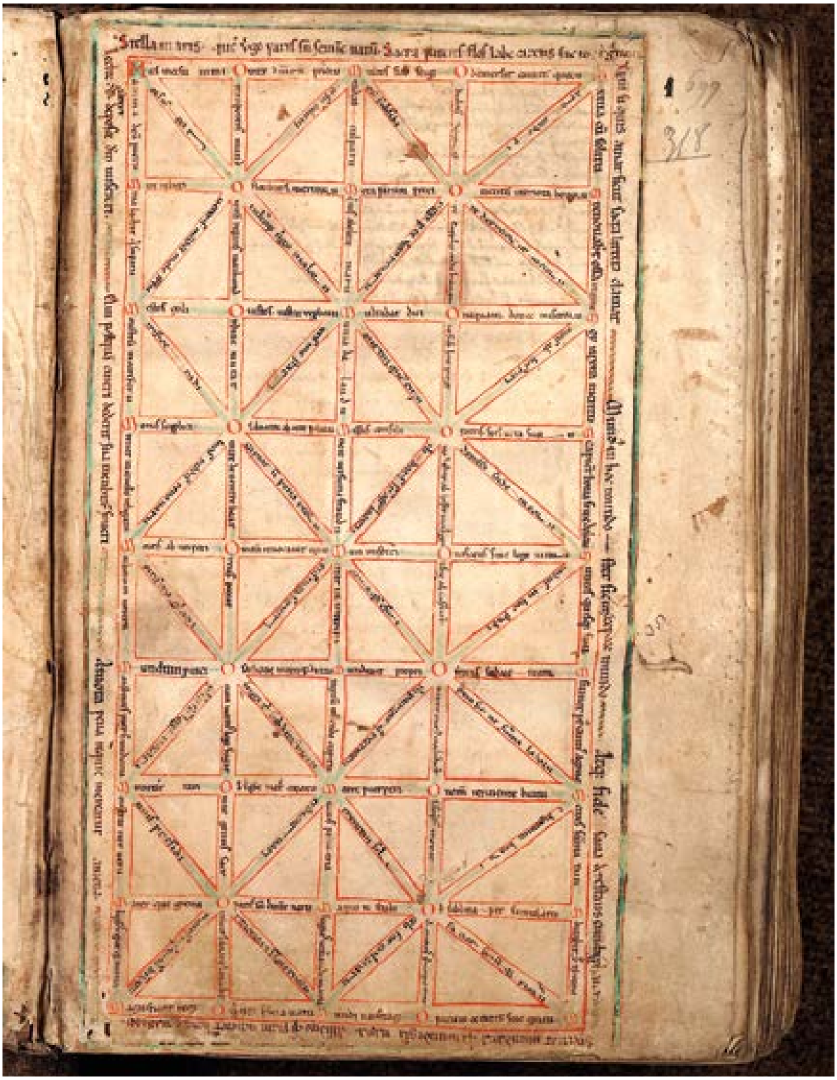
I am wondering about the role technology plays in the construction of these fantasies or imaginaries. I mean, on the one hand technology is the object of them; on the other, it is the subject, in the sense that it creates new imaginaries, expectations, hopes, and fears to cope with reality. Maybe we need, as you said, an imaginary of the master algorithm even more than the master algorithm itself!

Images

Image 1. The penultimate full-page miniature in the Rothschild Canticles (f. 104r) displays a peculiar version of a maxim that had a long tradition in medieval philosophy: *Centrum meum ubique, circumferentia nusquam.*

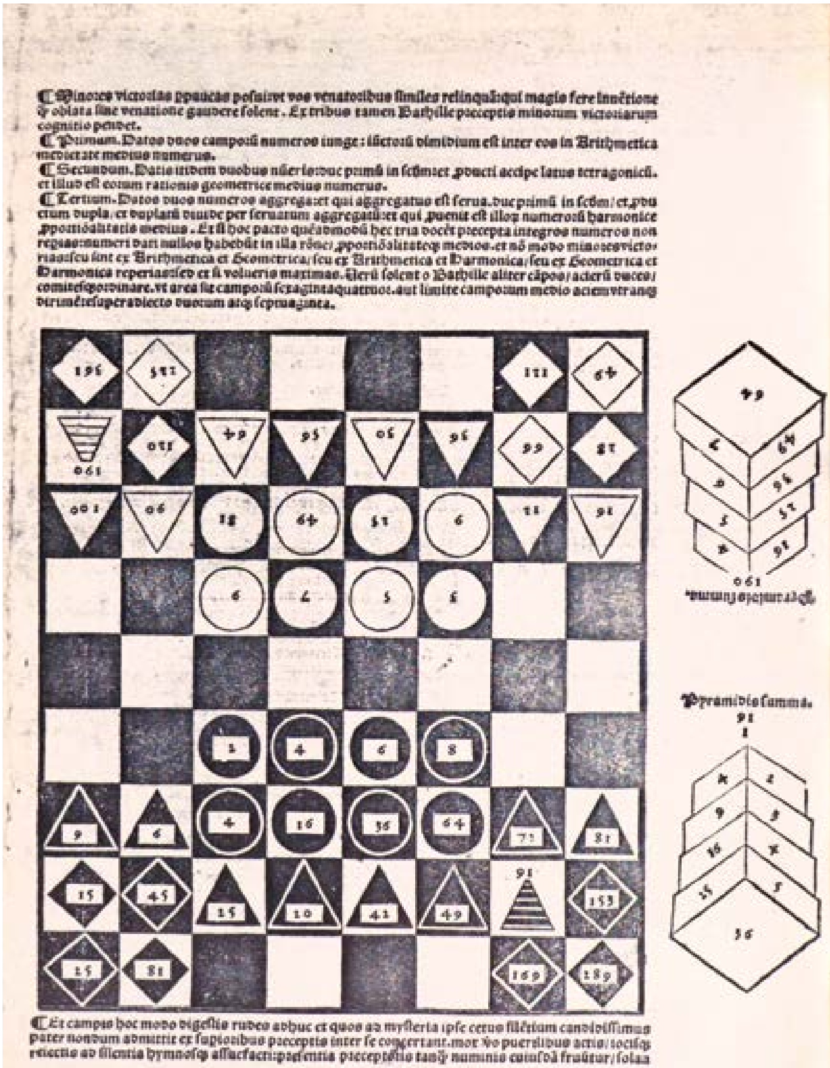
Rothschild Canticles, [ca. 1300]. General Collection, Beinecke Rare Book and Manuscript Library, Yale University. Beinecke MS 404, f.104r. Retrieved from https://collections.library.yale.edu/catalog/2002755 Accessed 09 Dec. 2021. Image in Public Domain.

Image 2. This diagram constitutes a preface of Isidore's enciclopedic work about words meaning. It does so in ways that are pictorial as well as poetic.



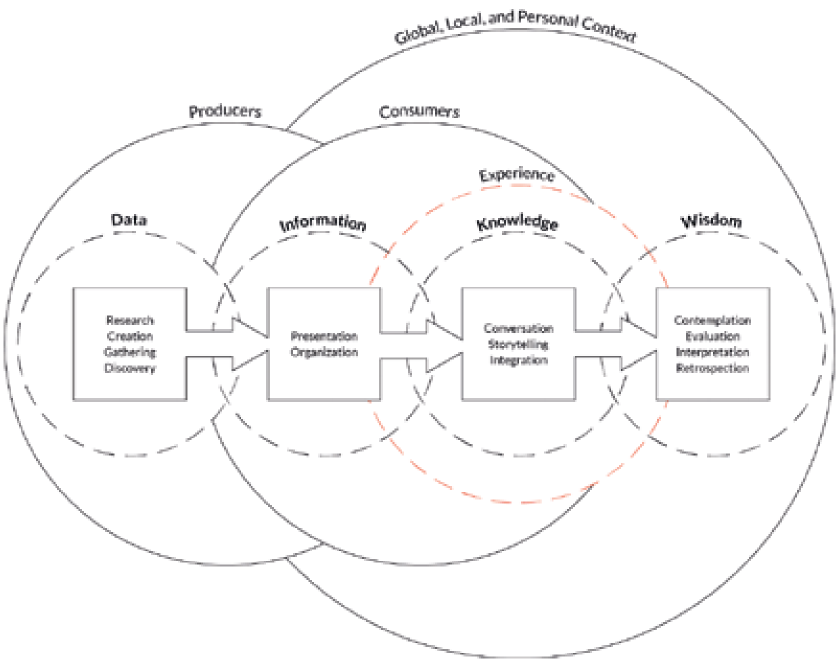
Isidorus Hispalensis, *Etymologiarum libri XX*, 3rd quarter of 12th century. Douai, Bibl. mun., ms. 0318, f. 1r. Retrieved from *https://bvmm.irht.cnrs.fr/consult/consult.php?VUE_ID=1305797* Accessed 13 Jan. 2022. Printed with permission.

Image 3. Rithmomachia was a board game, designed to teach and practice the principles of Boethian arithmetics.



Jordanus Nemorarius, *In hoc opere contenta. Arithmetica decem libris demonstrata; Musica libris demonstrata quattuor ; Epitome iin libros arithmeticos diui Seuerini Boetii; Rithmomachie ludus qui et pugna numerorum appellatur, Jacobus Faber Stapulensis (Ed.)*, 1496. Paris, Joannes Higmanus et Volgangus Hopilius, f. 144r.

Image 4. The central workflow shows how data is transformed into wisdom, passing through the forms of information and knowledge. The diagram, called Understanding Spectrum, stresses how knowledge is shaped by user experience.



Understanding Spectrum
Redesigned by Dario Rodighiero from the original version by Nathan Shedroff ([1994] 1999).

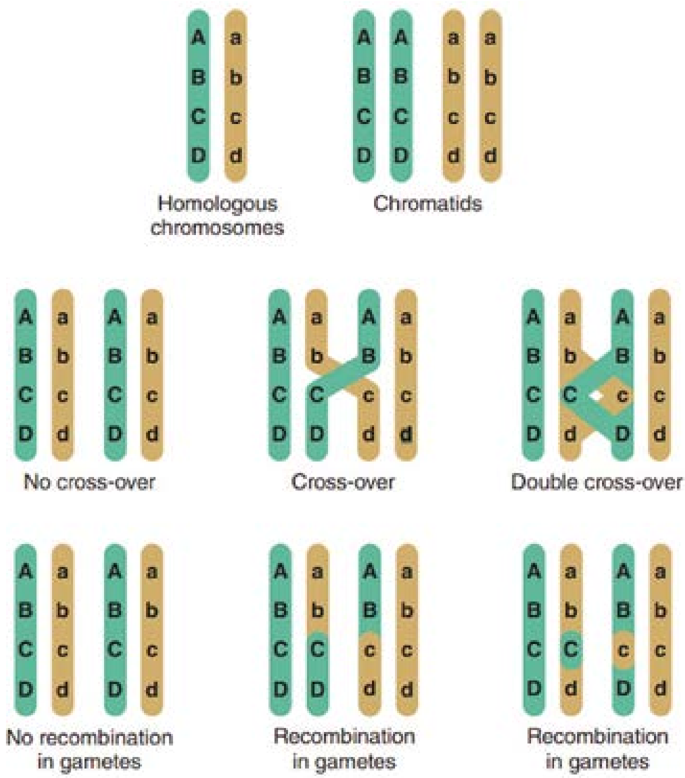
Image 5. Educational diagram composed by Andrea Bacci and executed by Natale Bonifacio in 1581. *Ars Memorativa* contributed to bring information design into diagrammatic layouts like this one, which presents the cognitive processes of the human being as part of the larger cosmographical system.

Adrianus Hubertus, *Ordo Universi et humanarum scientiarum prima monumenta,* Engraving, 1585 AD. Bibliothèque nationale de France, reserve QB-201 (170)-FT4. Retrieved from https://gallica.bnf.fr/ark:/12148/btv1b55002353x. Accessed 12 Aug. 2021. Image in Public Domain.

Image 6. Clay tablet from Uruk recording the allocation of beer, dating 3000 BC and preserved by the British Museum. Contrarily to the digital objects that have already disappeared, some clay tablets have lasted more than five millenia to come to our days.

Late Uruk clay tablet; record of beer; impressed with five different types of numerical symbol. 3000 BC © The Trustees of the British Museum, Asset number 121754001. Printed with permission.

Image 7. Crossing-over and genetic recombination.



Jameson, J.L., Fauci, A., Kasper, D., Hauser, S., Longo, D., Loscalzo, D. *Harrison's Principles of Internal Medicine*, 20th Edition (volumes 1 and 2). 2018 AD. McGraw-Hill Education, New York. p. 3352. Printed with the permission of the editors.

Image 8. Chart of rounded glass flasks with urine of different colours. Medieval physicians often made diagnoses by examining the appearance of their patients' urine, holding flasks of urine against a light source, assessing the colour and viscosity and trying to see fragments. Familiar with the theory of the admixtures of humours and a urine analysis chart, they had to perceive if the urine showed any problems and which ones.

John Somer & Robert Grosseteste. *Physician's Folding Almanac*. c. 1406 AD, British Library, Harley Collection, Harley 5311, Section F. Retrieved from: https://www.europeana.eu/pt/item/9200397/BibliographicResource_3000126283294. Accessed 04 Jan. 2022. Image in Public Domain.

Image 9. The first figure is the Elemental Figure (*figura elementalis*). In this figure Ramon Llull represents the four elements and the dynamics of their relationship. Llull assigns to the four elements the first letters of the alphabet, to be able to express natural phenomena with combination formulas (AB, CD, etc.), from which one sees the connection between natural philosophy and Llull's art. The *figura elementalis* already appears in the pre-Art phase, namely in 1274 in the *Ars compendiosa inveniendi veritatem*. It is described as part of figure T. This fact is very important, because it gives the Elemental Figure a gnoseological function in the art of Llull. The figure of fire can be likened to the peppercorn which, according to Llull's natural philosophy, is found in the fourth degree of heat.

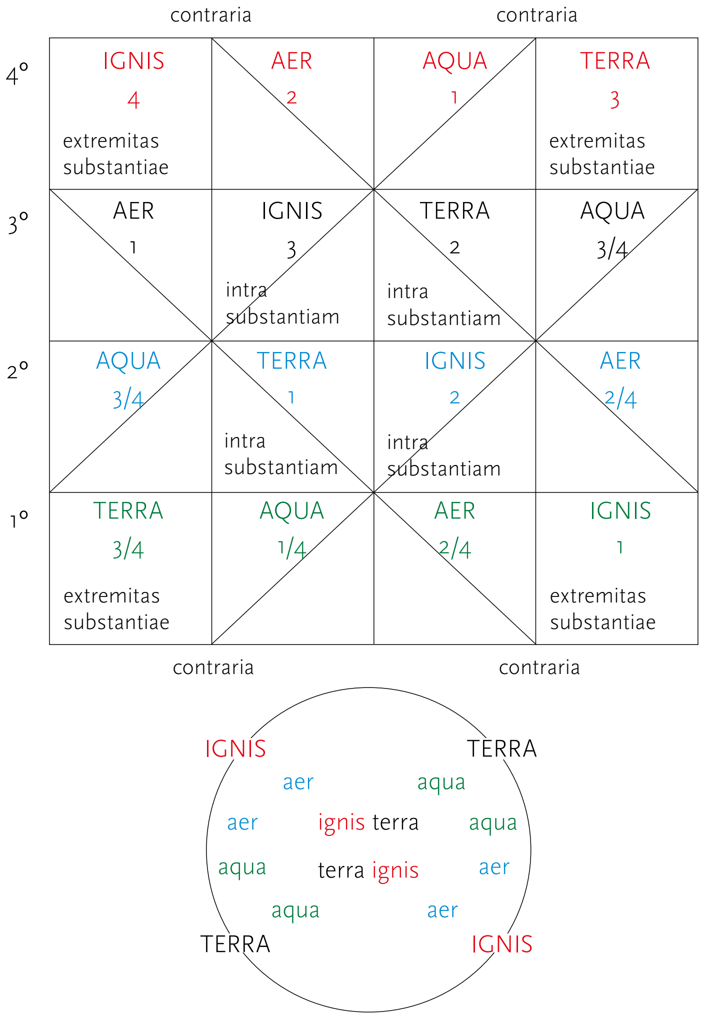


Figura elementalis – diagrams designed by Carla Compagno.

Image 10. The second figure is a three-dimensional reconstruction of the Figure of the fourth heat degree (*figura quarti gradus caloris*), that appears in the *Liber de nova geometria* (1299). In this figure, the four degrees of fire are represented by the letter A, the three degrees of earth by the letter C, the two of air by the letter B, and the sole degree of water by the letter D. This seems an attempt to represent the elemental particles within material substance. After a brief introduction to Llull's system of diagrams, here we will focus on the *figura elementalis* in the ars, and on the evolution of its meanings in Llull's works.

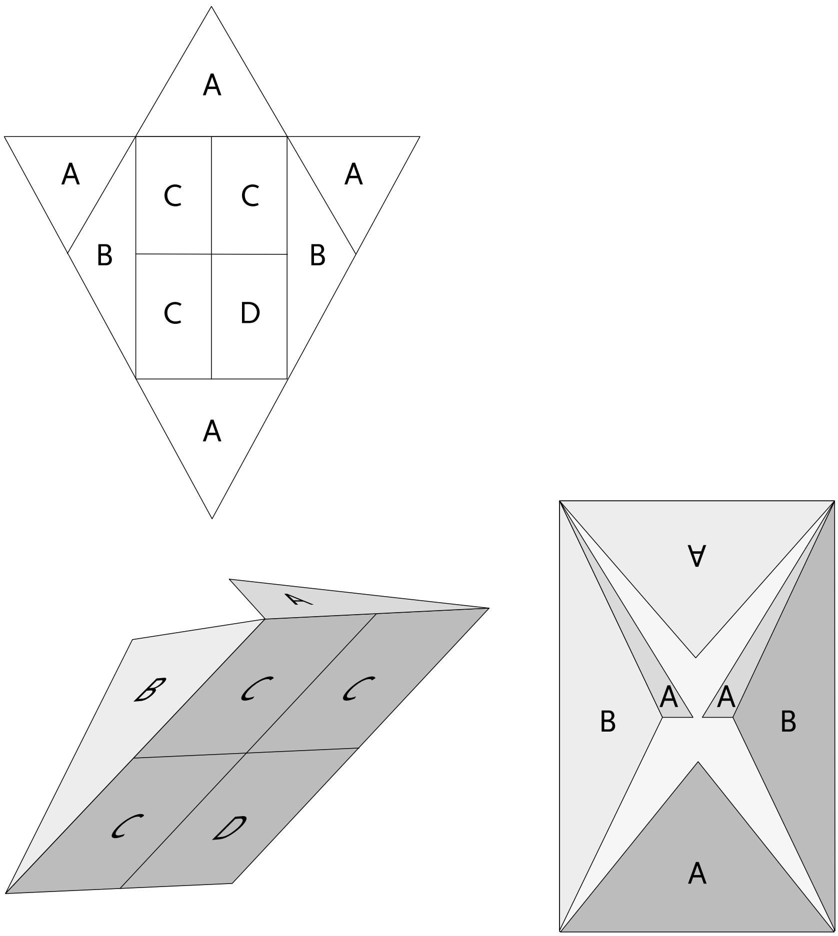
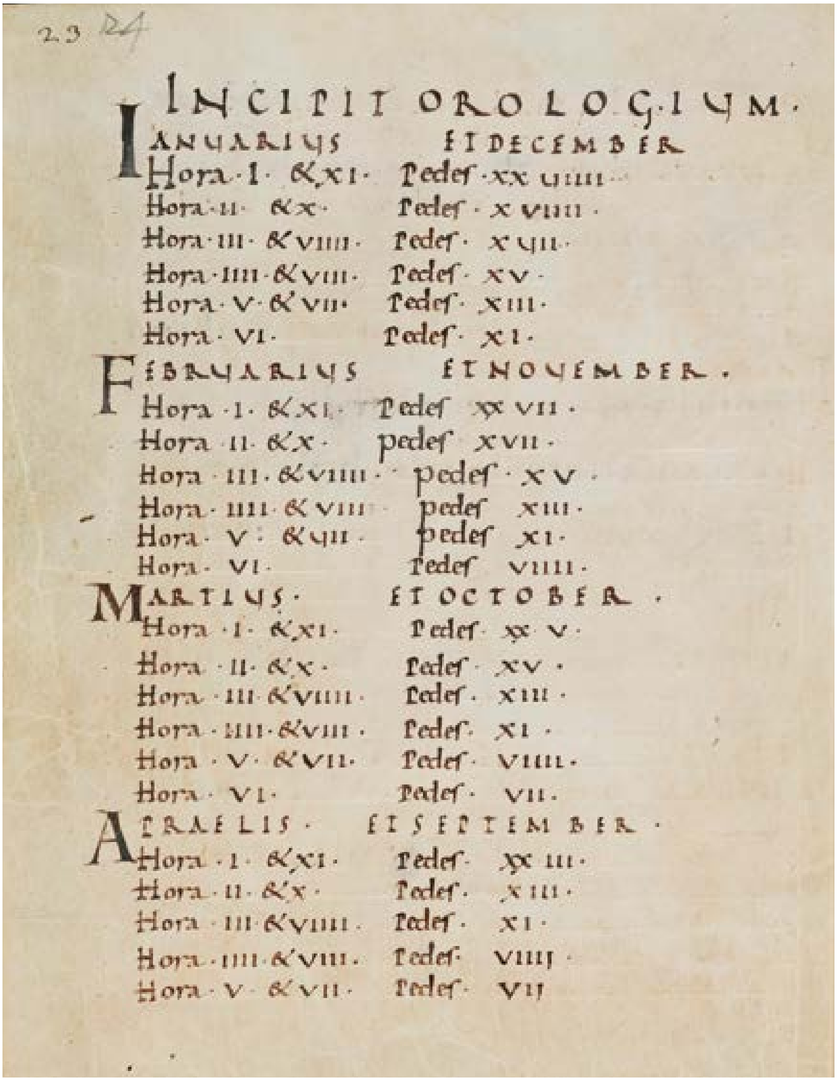


Figura quarti gradus caloris – diagrams designed by Carla Compagno.

Image 11. Two column table with the concordance of months and hours of daylight, with the lengh of the shadow of the Sun in *pedes* occupaying pages 24 and 25. Possibly written in the second half of the 9th century for Grimald of St. Gall, with the contribution of c. 40 scribes/hands.



23 24

INCIPIT OROLOGIUM.

IANUARIUS	ET DECEMBER
Hora. I. & XI.	Pedes. XX VIIII
Hora. II. & X.	Pedes. X VIIII
Hora. III. & VIIII.	Pedes. XVII
Hora. IIII. & VIII.	Pedes. XV
Hora. V. & VII.	Pedes. XIII
Hora. VI.	Pedes. XI
FEBRUARIUS	ET NOVEMBER
Hora. I. & XI.	Pedes XX VII
Hora. II. & X.	pedes XVII
Hora. III. & VIIII.	pedes. XV
Hora. IIII. & VIII	pedes XIII
Hora. V. & VII.	pedes XI
Hora. VI.	Pedes VIIII
MARTIUS	ET OCTOBER
Hora. I. & XI.	Pedes. XX V
Hora. II. & X.	Pedes. XV
Hora. III. & VIIII.	Pedes. XIII
Hora. IIII. & VIII.	Pedes. XI
Hora. V. & VII.	Pedes. VIIII
Hora. VI.	Pedes. VII
APRELIS	ET SEPTEMBER
Hora. I. & XI.	Pedes. XX III
Hora. II. & X.	Pedes. XIII
Hora. III. & VIIII.	Pedes. XI
Hora. IIII. & VIII.	Pedes. VIIII
Hora. V. & VII.	Pedes. VII

Miscellanea liturgica, historica, poetica. 9th century, St. Gallen, Stiftsbibliothek, Cod. Sang. 397, p. 24. Retrived from https://www.e-codices.ch/en/list/one/csg/0397. Accessed 09 Dec. 2021. Printed with permission.

Image 12. The diagram of the *Horologium Viatorum* is part of the codicological unit '*Excerpta ex Isidoro et Beda Venerabili, De cyclo paschali*'. It is a device used to learn the time based on the hours of daylight throughout the year, measuring the length of the shadow of the Sun, taking the human body as a gnomon.

Computational/scientific compilation with numerous tables, schematics. 9th/10th century, St. Gallen, Stiftsbibliothek , Cod. Sang. 459, fol. 141r. Retrieved from https://www.e-codices.ch/en/list/one/csg/0459. Accessed 09 Oct. 2021. Printed with permission.

Image 13. The three-part *Horologium* diagram, composed of two rotae for the *Sphere Lunae* and the *Sphere Solis* on top and a *Horologium Viatorum* bellow. Copied at the The Cathedral School of Laon, in Aisne, Hauts-de-France, in the first half of the 9th century.

Isidorus Hispalensis, De natura rerum, Etymologiae (lib. 3, interpol.); Computus (1-72r); Isidorus Hispalensis, Sententiae (73v-93). 9th century. Bibliothèque municipale Suzanne Martinet de Laon. ms. 422, fol. 53r. Retrieved from https://gallica.bnf.fr/ark:/12148/btv1b8492138z/f117.item. Accessed 09 Dec. 2021. Image in Public Domain.

Image 14. The River Nile, from *Kitāb Gharā'ib al-Funūn wa-Mulaḥ al-'Uyūn* (كتاب غرائب الفنون وملح العيون)

Kitāb Gharā'ib al-funūn wa-mulaḥ al-'uyūn (The Book of Curiosities of the Sciences and Marvels for the Eyes). late 12th or early 13th century, Oxford, Bodleian Library MS. Arab. c. 90, f. 42a. Retrieved from https://digital.bodleian.ox.ac.uk/objects/748a9d50-5a3a-440e-ab9d-567dd68b6abb/ Accessed 8 Oct. 2021. Creative Commons licence CC-BY-NC 4.0.

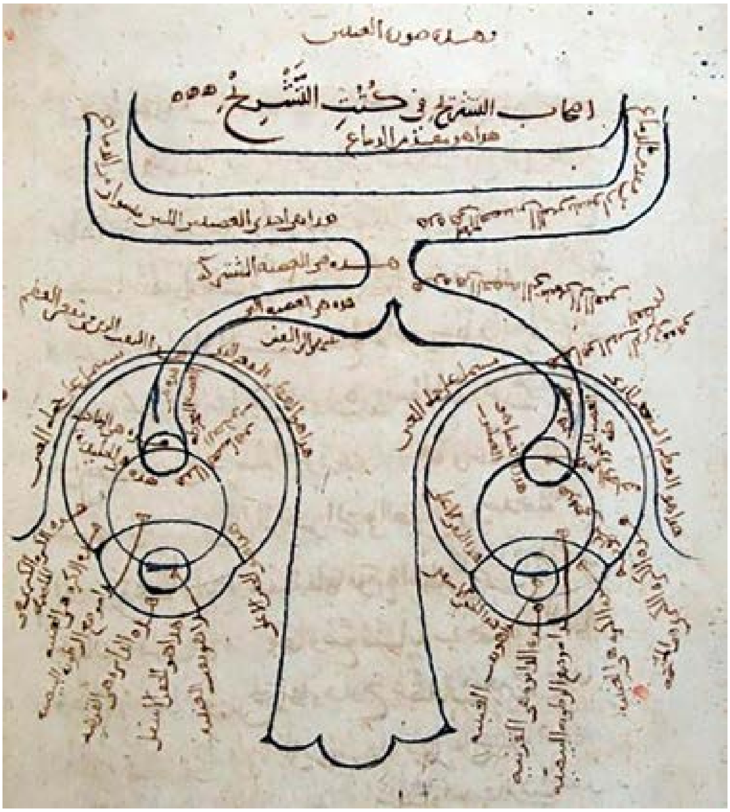
Image 15. The River Tigris, from *Kitāb Gharā'ib al-Funūn wa-Mulaḥ al-'Uyūn* (كتاب غرائب الفنون وملح العيون)

Kitāb Gharā'ib al-funūn wa-mulaḥ al-'uyūn (The Book of Curiosities of the Sciences and Marvels for the Eyes), author unknown, late 12th or early 13th century, Oxford, Bodleian Library MS. Arab. c. 90, f. 43a. Retrieved from https://digital.bodleian.ox.ac.uk/objects/748a9d50-5a3a-440e-ab9d-567dd68b6abb/ Accessed 8 Oct. 2021. Creative Commons licence CC-BY-NC 4.0.

Image 16. The River Indus, from *Kitāb Gharā'ib al-Funūn wa-Mulaḥ al-'Uyūn* (كتاب غرائب الفنون وملح العيون)

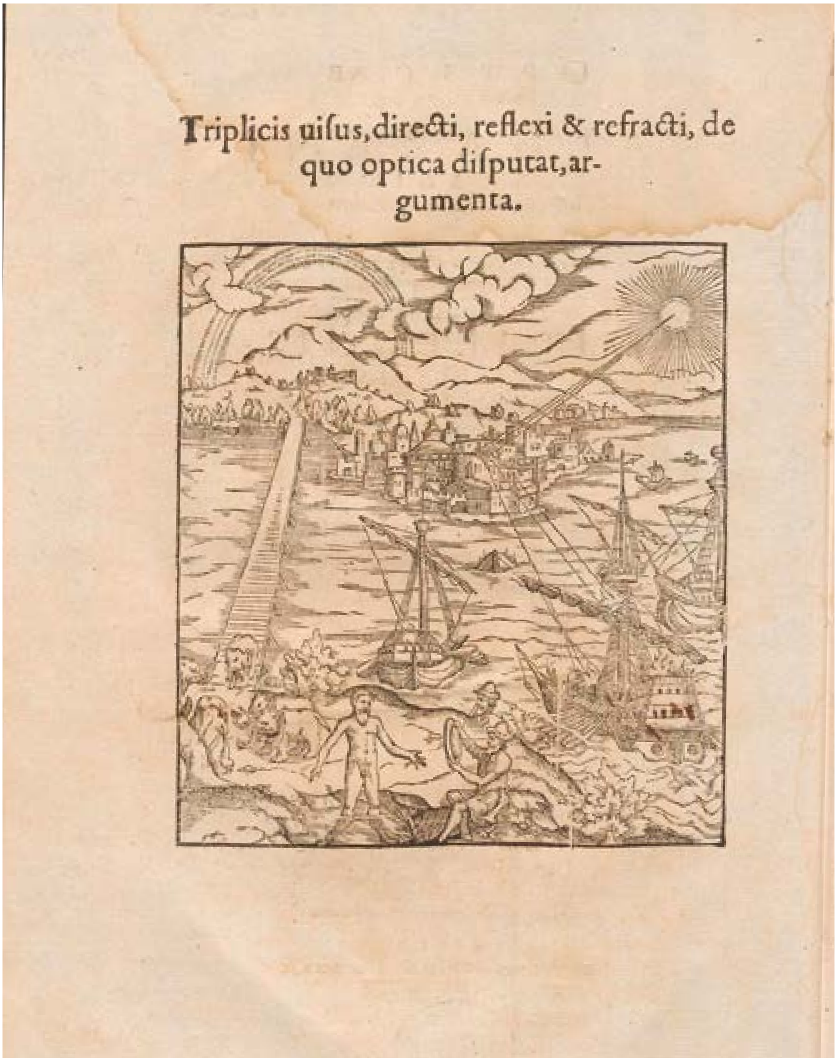
Kitāb Gharā'ib al-funūn wa-mulaḥ al-'uyūn (The Book of Curiosities of the Sciences and Marvels for the Eyes), author unknown, late 12th or early 13th century, Oxford, Bodleian Library MS. Arab. c. 90, f. 43b Retrieved from https://digital.bodleian.ox.ac.uk/objects/748a9d50-5a3a-440e-ab9d-567dd68b6abb/ Accessed 8 Oct. 2021. Creative Commons licence CC-BY-NC 4.0.

Image 17. 'A diagram of the eye'.



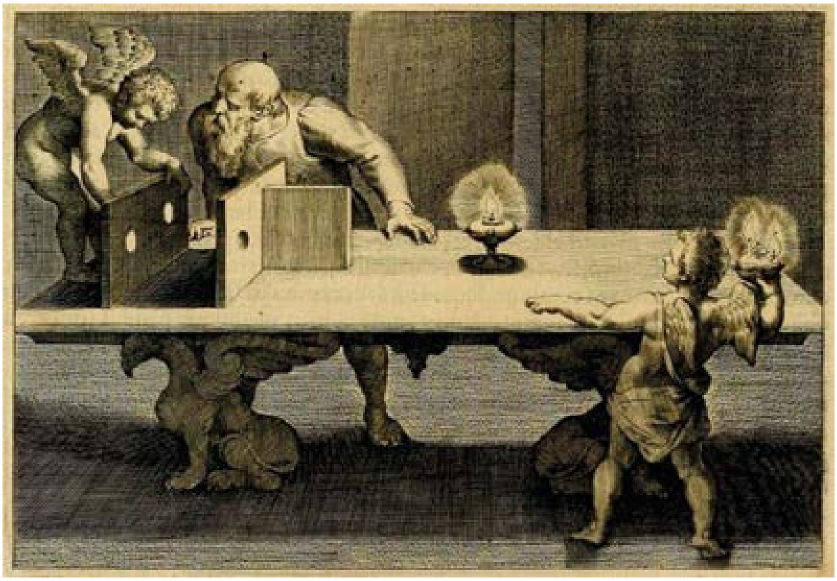
Kitāb al-Manāẓir, Ibn al-Haytham, 11th century, Süleymaniye Mosque Library, Istanbul, MS Fatih 3212, vol. 1, fol. 81b. Retrieved from https://en.wikipedia.org/wiki/Book_of_Optics#/media/File:Alhazen1652.png Accessed 29 Oct. 2021. Image in Public Domain.

Image 18. Frontispiece of the *Opticae Thesaurus* edition by Friedrich Risner, of the first printed Latin translation of Ibn al-Haytham's *Kitāb al-Manāẓir*.



Ibn-al-Haiṯam, Witelo, Risner, F., & Caccia, F. *Opticae thesaurus. Alhazeni Arabis libri septem, nunc primum editi...*, 1572, Basel. ETH Bibliothek RA Rara Rar 9065 VD16 H 693, f. a1v. Retrieved from: https://www.e-rara.ch/zut/content/titleinfo/2886167. Accessed 10 Dec. 2021. Image in Public Domain.

Image 19. Vignette for the *Opticorum libri sex* of François d'Aguilon



Peter Paul Rubens, *Opticorum libri sex (*of François d'Aguilon), 1613. Antwerp. Retrieved from https://rkd.nl/nl/explore/images/262882. Accessed 29 Oct. 2021. Image in Public domain.

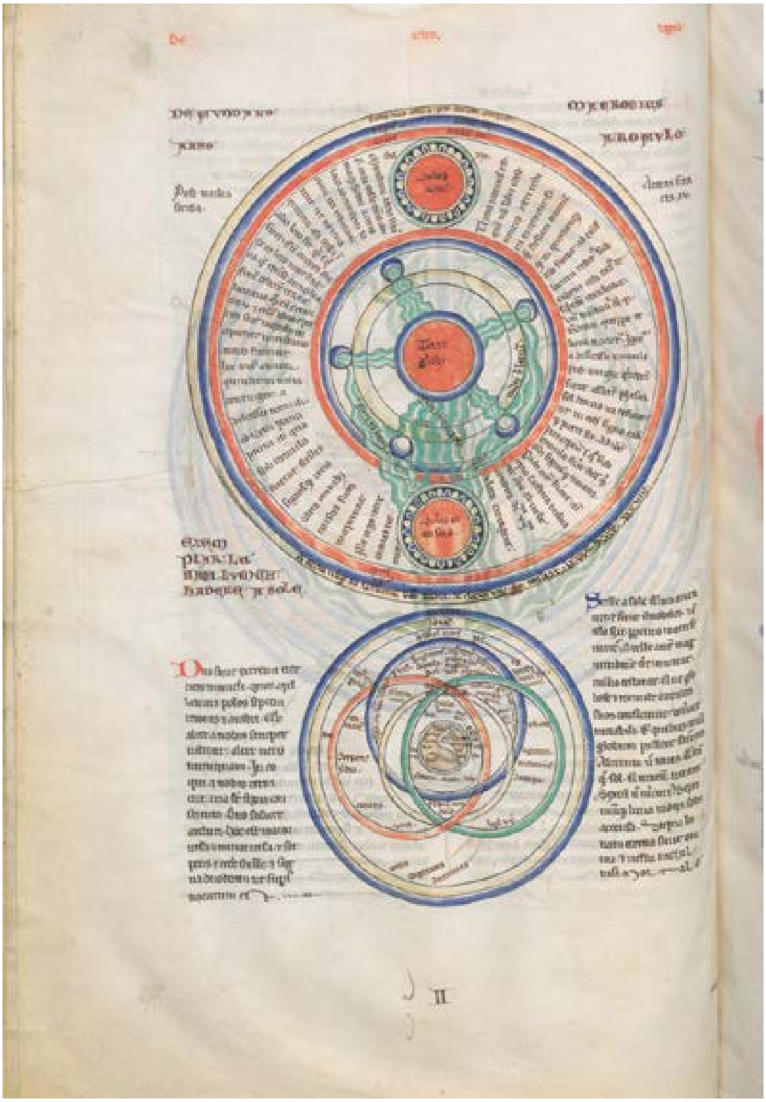
Image 20. Frontispiece of Johannes Hevelius' *Selenographia*

Johannes Hevelius, *Selenographia, sive Lunae descriptio...*, 1647. Gdańsk. Frontispiece. Bibliothèque nationale de France, département Réserve des livres rares, RES G-V-508 (1). Retrieved from https://gallica.bnf.fr/ark:/12148/bpt6k322596j/f7.item# Accessed 10 Dec.t. 2021. Image in Public Domain.

Image 21. This set of diagrams represents astronomical phenomena such as eclipses. Also, it displays the colors of each planetary sphere that resemblances the rainbow's colors.

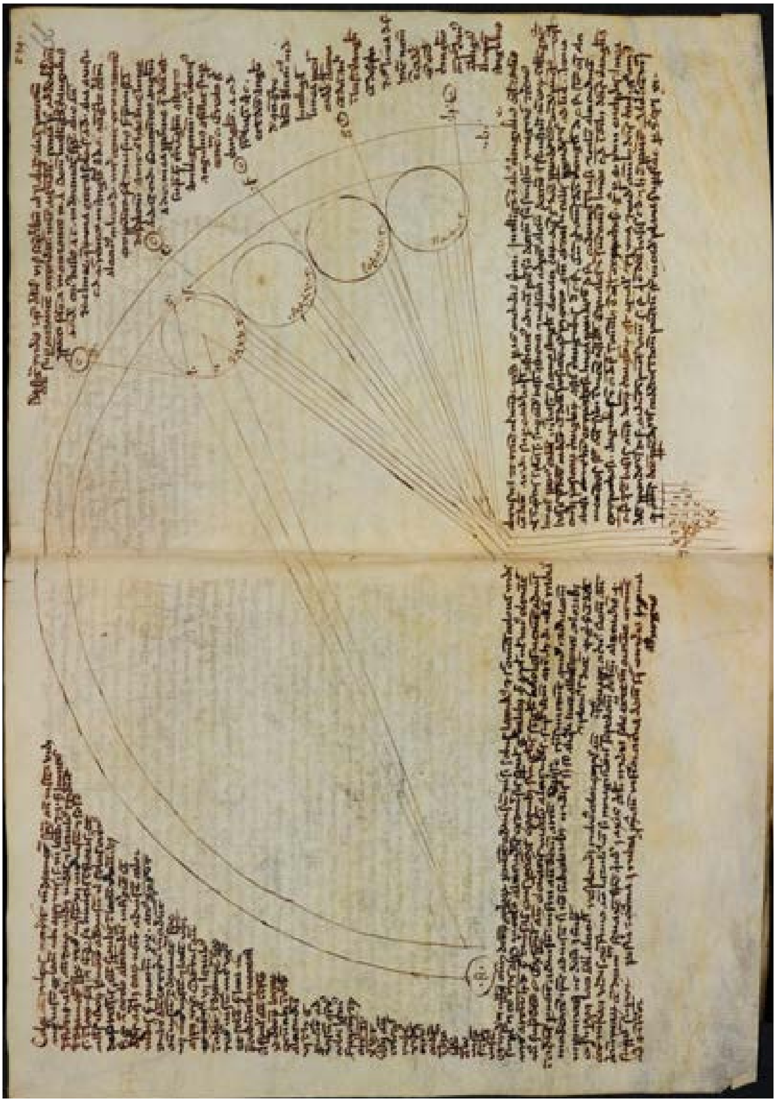
Lambertus de Sancto Audomaro, *Liber Floridus*. 13th century. Ghent University Library, Ghent MS. 92 f. 225v. Retrived from https://lib.ugent.be/en/catalog/rug01:000763774. Accessed 10 Dec. 2021. Image in Public Domain.

Image 22. That image reproduces the apparent disposition of the cosmic phenomena and the planetary light source – the sun, resembling as well the order of rainbow's colors.



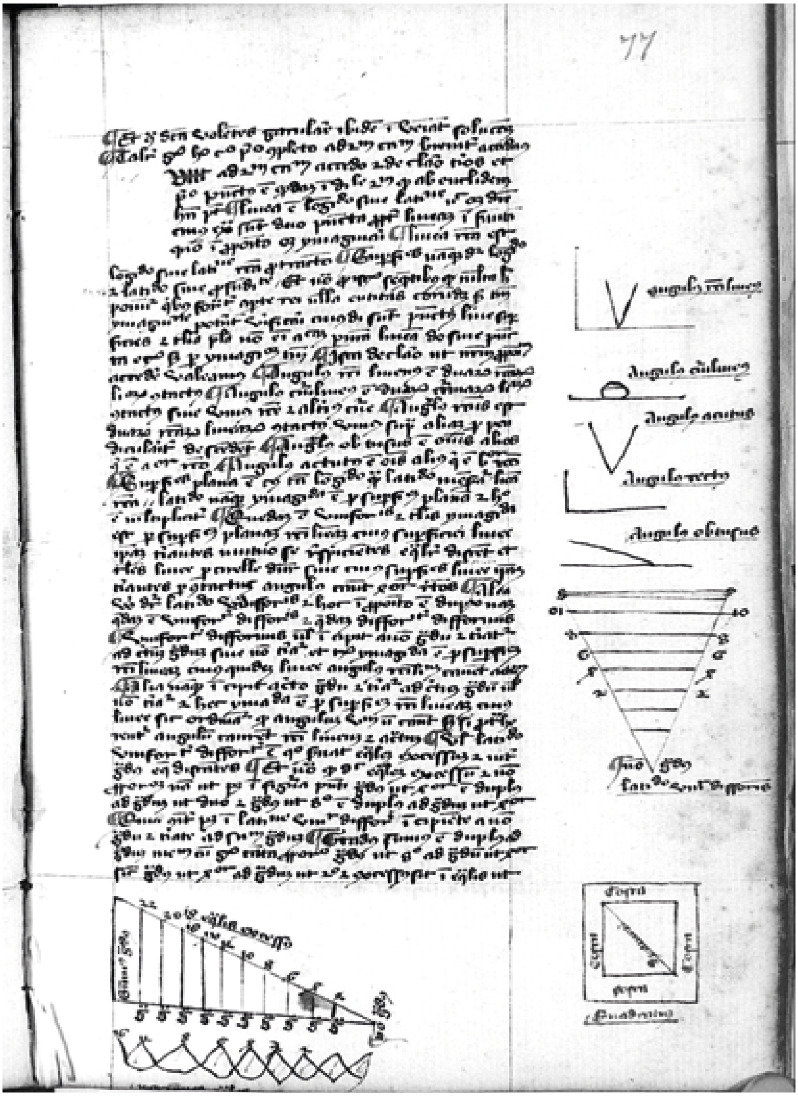
Lambertus de Sancto Audomaro, *Liber Floridus*. c. 1250 century. Bibliothèque nationale de France. Département des Manuscrits. Latin 8865, f. 56v. Retrieved from https://gallica.bnf.fr/ark:/12148/btv1b6000541b/f124.item. Accessed 10 Dec. 2021. Image in Public Domain.

Image 23. The minimal process of reflection and diffraction is represented in De iride trough a complex series of diagramas in which the optical phenomena is "enlarged".



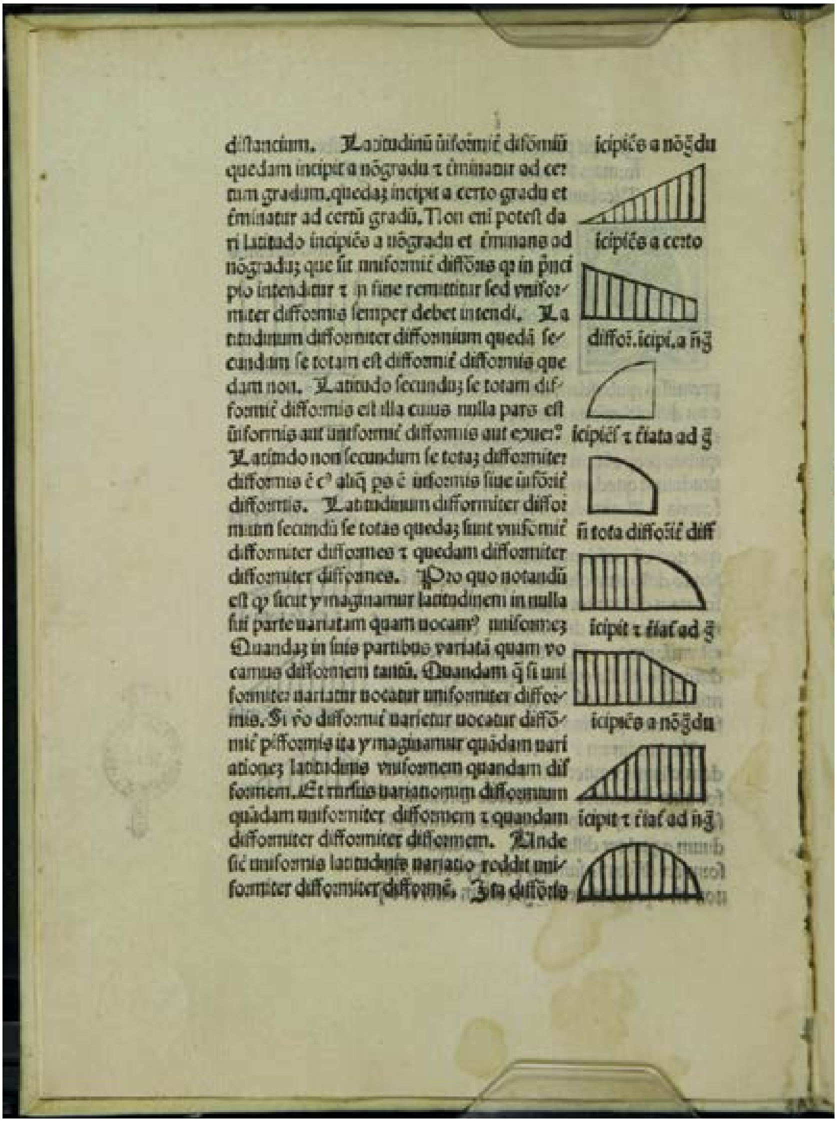
Theodoric of Freiberg, *De iride et radialibus impressionibus*. Leipzig, Universitätsbibliothek Leipzig, Ms 512, ff. 65v-66r. Retrived from http://www.manuscripta-mediaevalia.de/?xdbdtdn!%22obj%20 90792817,T%22&dmode=doc#|4. Accessed 10 Dec. 2021. Image in Public Domain.

Image 24. A manuscript page of Jacob of Naples' treatise *De perfectione specierum* with diagrams produced by the scribe clarifying some passages of the text.



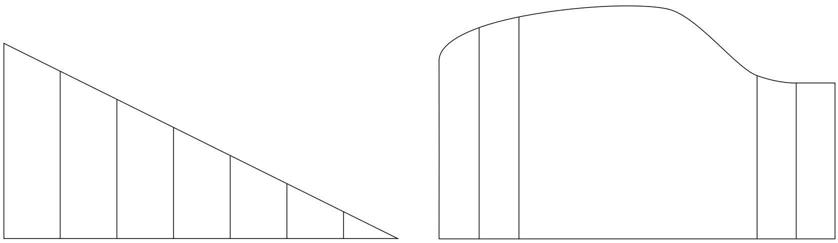
Bayerische Staatsbibliothek München, Clm 26838, fol. 77r. Retrived from https://www.digitale-sammlungen.de/en/view/bsb00106176?page=158. Accessed 10 Dec. 2021. Creative Commons licence CC-BY-NC 4.0.

Image 25. The *Tractatus de latitudinibus formarum* is conveyed in more than fifty manuscripts and four old printed editions. During the Middle Ages this text was wrongly attributed to Nicole Oresme, as in this copy, appeared in Padua, 1482 (and again in 1486) including also the questions on the same text by the famous philosopher Biagio Pelacani da Parma. The diagrams on the right margin visualize the definitions and divisions of the different latitudes explained in the text.



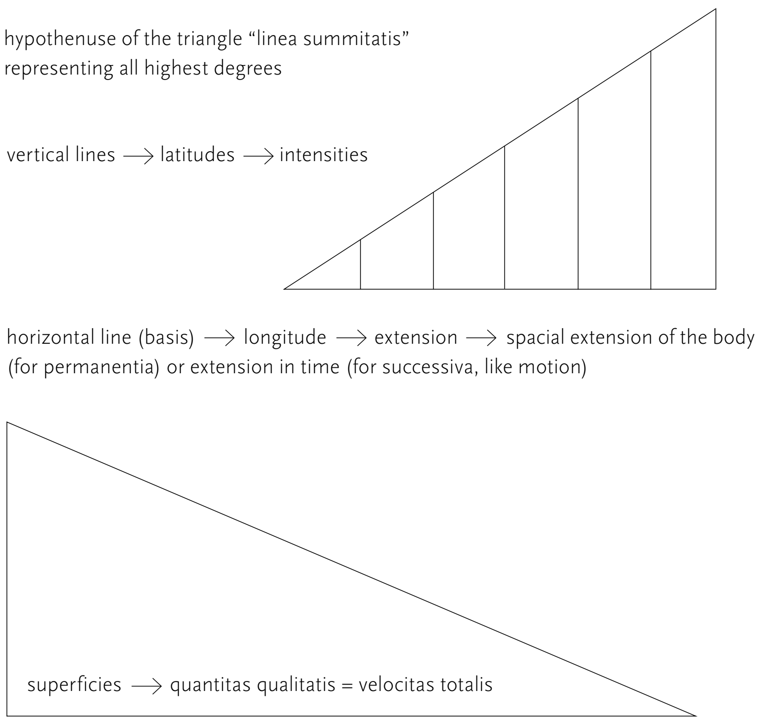
Tractatus de latitudinibus formarum, ed. 1482. Padua. Biblioteca Nazionale Centrale di Firenze, Pal. E.6.3.142, p.2. Printed with permission.

Image 26. Left: a *latitudo uniformter difformis*. Right: *a latitudo difformiter difformis*. Both as represented and explained in Jacob of Naples's *De perfectione specierum*.



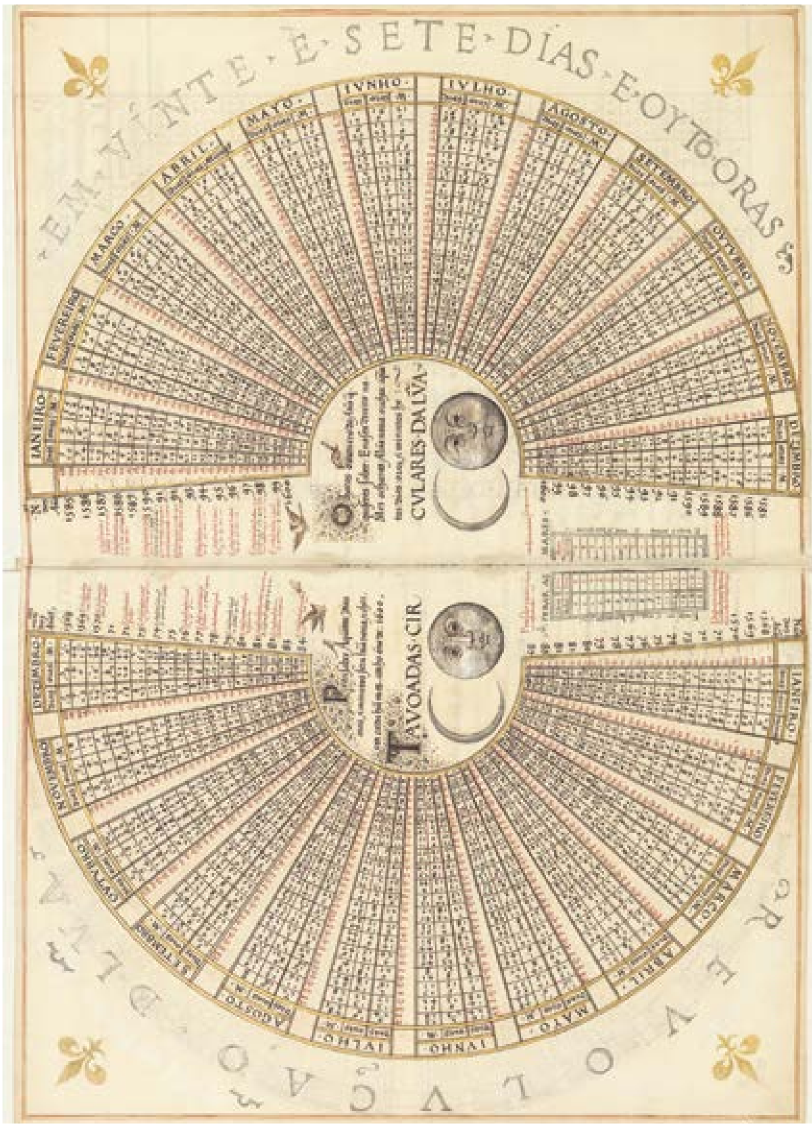
Diagrams designed by Daniel A. Di Liscia.

Image 27. Two Oresmian configurations representing a *qualitas difformiter difformis*. For the sake of brevity an arrow "⟶" was used, making clear to which physical/ontological notion is referred by a given geometrical term. Thus, for instance, the geometrical term "superficies" refers to the "total quality" or the "quantity of a quality." For motions, the quantity of the quality can be called the "total velocity" which, in turn, only in the special case of local motion expresses the covered space in an interval of time.



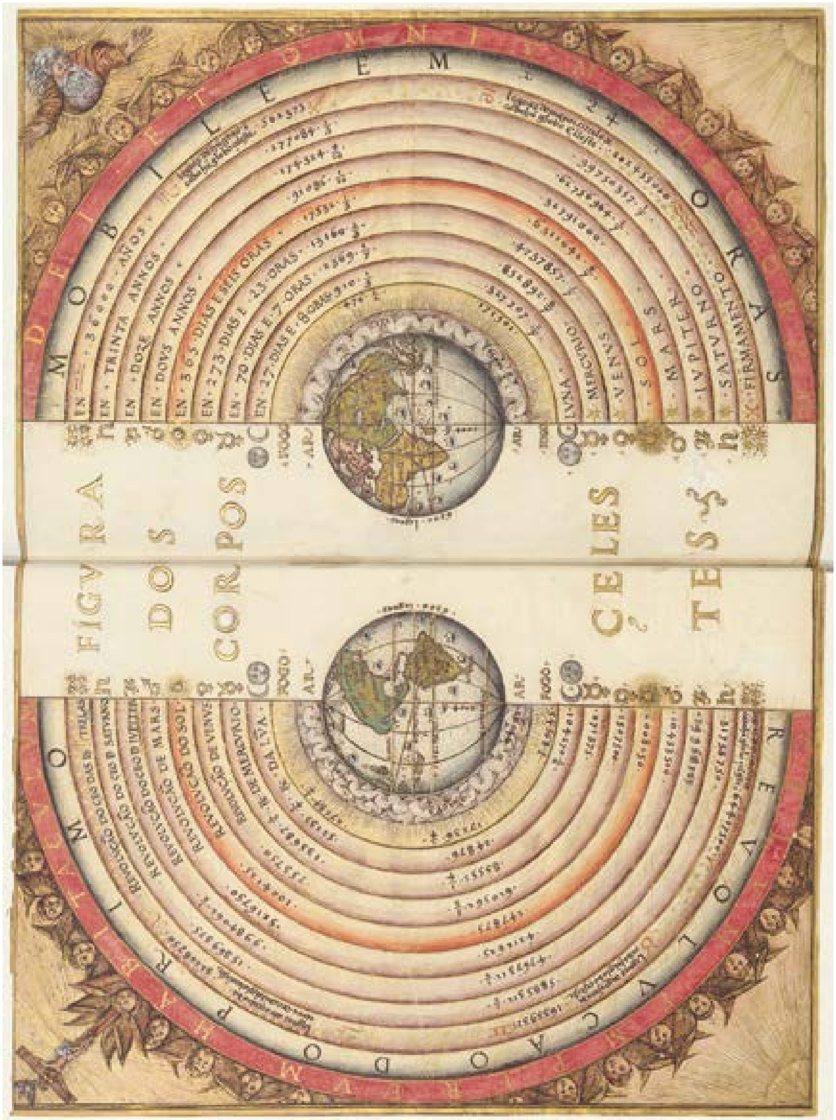
Diagrams designed by Daniel A. Di Liscia.

Image 28. *Revolução da Lua em vinte e sete dias e oyto oras, de 1568 a 1600* [Moon Revolution in twenty-seven days and eight hours, from 1568 to 1600].
(bifolio size: 33,5 x 47,0 cm)



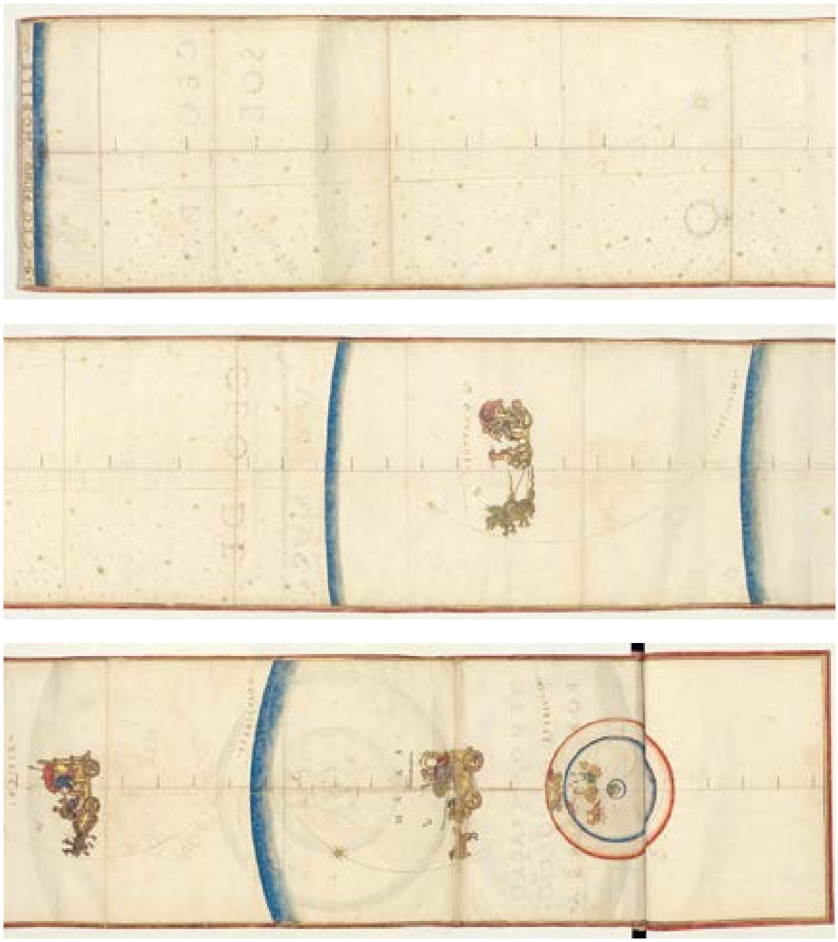
Bartolomeu Velho, *Cosmographia*, 1568. Paris, Bibliothèque nationale de France, département Cartes et plans, GE EE-266 (RES), ff. 4v–5r. Retrived from https://catalogue.bnf.fr/ark:/12148/cb416166390. Accessed 10 Dec. 2021. Image in Public Domain.

Image 29. *Figura dos corpos çelestes* [Scheme of the Celestial Bodies]. (bifolio size: 33,5 x 47,0 cm)



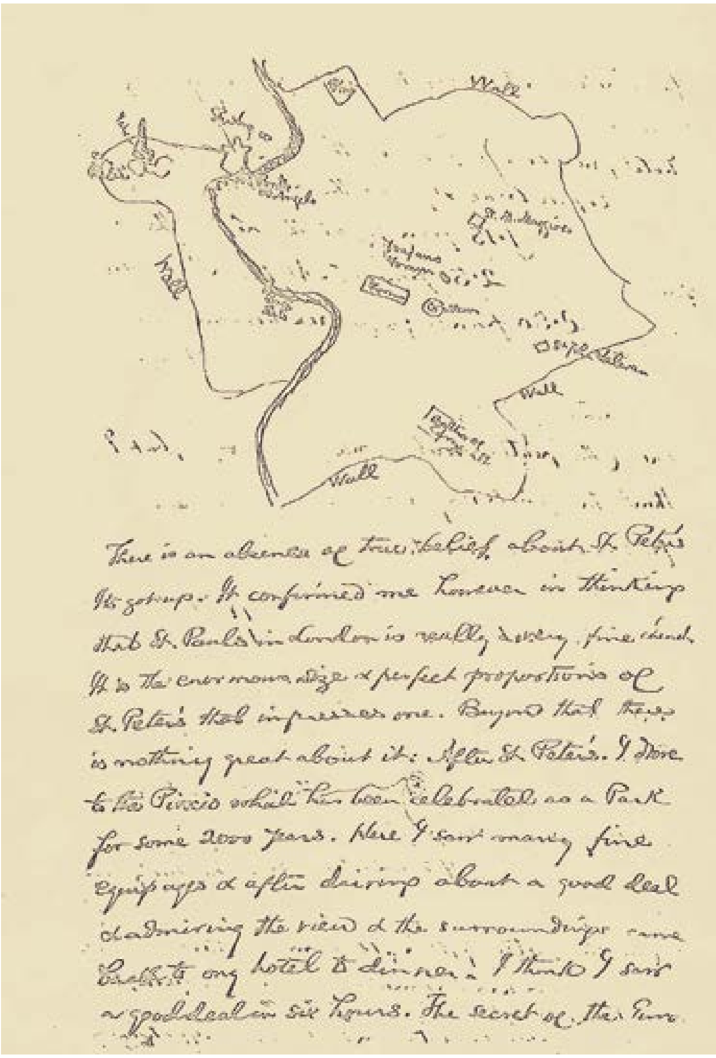
Bartolomeu Velho, *Cosmographia,* 1568. Paris, Bibliothèque nationale de France, département Cartes et plans, GE EE-266 (RES), ff. 10v–11r. Retrived from https://catalogue.bnf.fr/ark:/12148/cb416166390. Accessed 10 Dec. 2021. Image in Public Domain.

Image 30. *Demonstração em proporção* (do Universo) *por legoas, da Terra ao Primo Mobilem* [Proportional Demonstration (of the Universe) in "legoas" from Earth to *Primo Mobilem*].
(foldable bifolio, size: 33,5 x 299,5 cm)



Bartolomeu Velho, *Cosmographia*, 1568. Paris, Bibliothèque nationale de France, département Cartes et plans, GE EE-266 (RES), ff. 21v–22r. Retrived from https://catalogue.bnf.fr/ark:/12148/cb416166390. Accessed 10 Dec. 2021. Image in Public Domain.

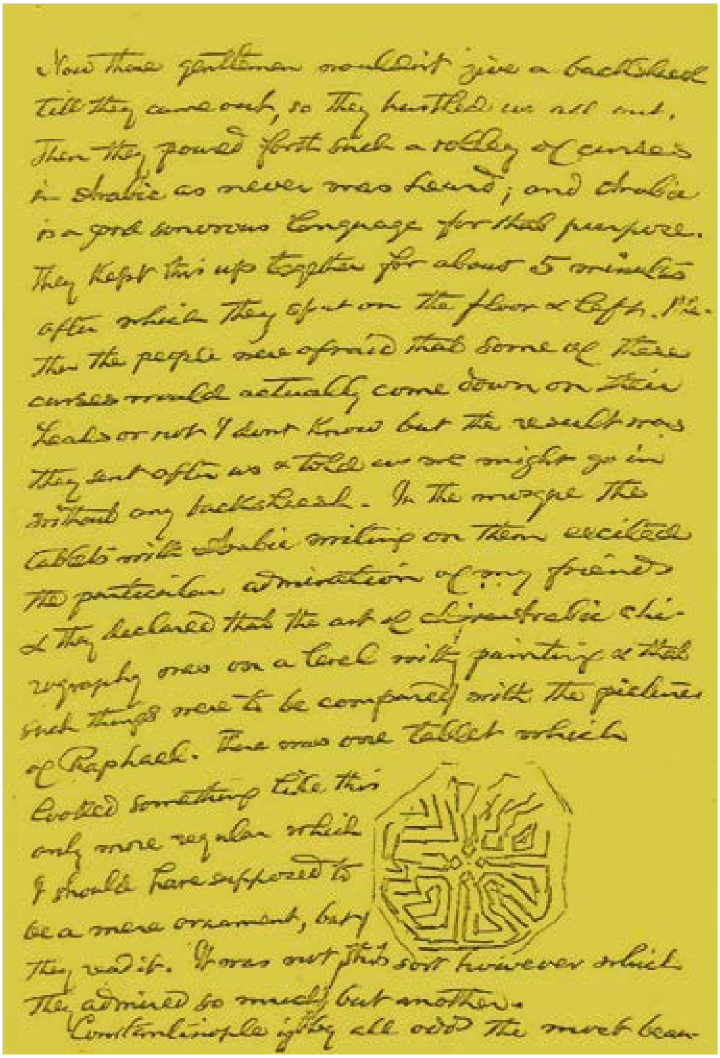
Image 31. [Map of the route that Peirce followed in Rome.]



There is an absence of true belief about St. Peter's
Its get-up. It confirmed me however in thinking
that St. Paul's in London is really a very fine church.
It is the enormous size & perfect proportions of
St. Peter's that impresses one. Beyond that there
is nothing great about it. After St. Peter's I drove
to the Pincio which has been celebrated as a Park
for some 2000 years. Here I saw many fine
equipages & after driving about a good deal
& admiring the view & the surroundings came
back to my hotel to dinner. I think I saw
a good deal in six hours. The secret of the [illegible]

Peirce, Charles S., *Charles Sanders Peirce papers*, 1787-1951. 1870 October 14. Houghton Library, Harvard University, Cambridge, Mass. MS *Am* 1632 (L 341), p. 5. Image in Public Domain.

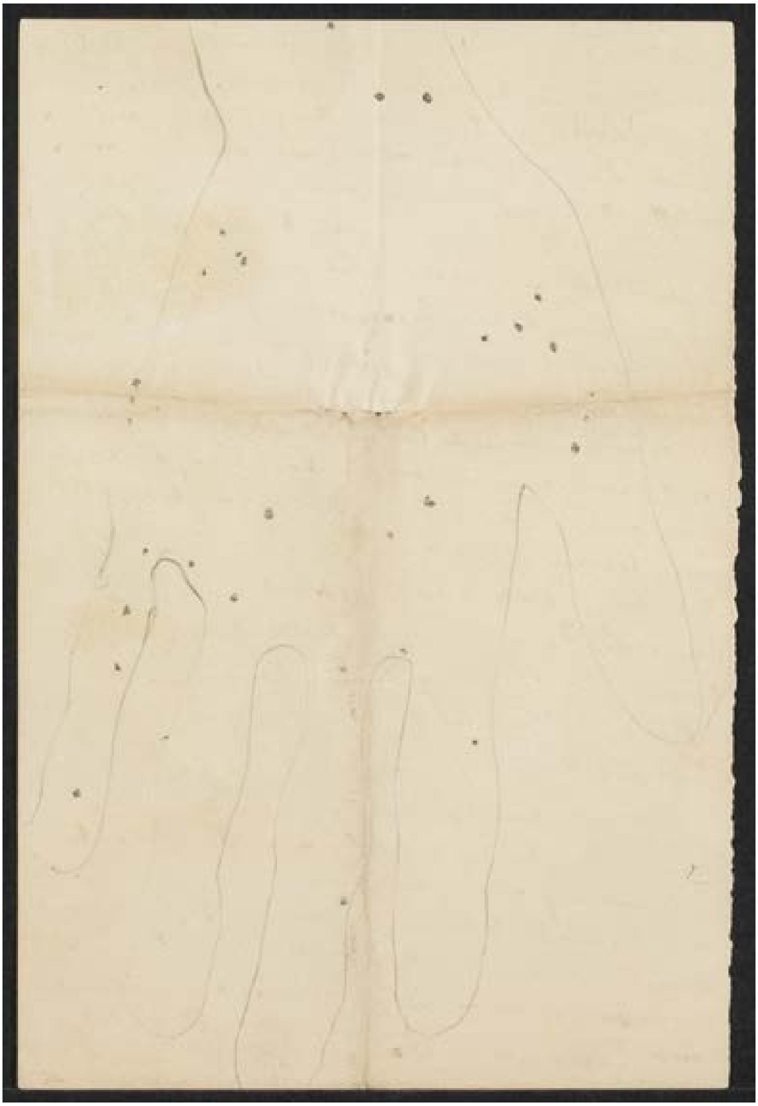
Image 32. [Tablet covered with Arabic script in the mosque of Hagia Sophia.]



Now these gentlemen wouldn't give a backsheesh
till they came out, so they hustled us all out.
Then they poured forth such a volley of curses
in Arabic as never was heard; and Arabic
is a good sonorous language for that purpose.
They kept this up together for about 5 minutes
after which they spat on the floor & left. The-
Then the people were afraid that some of these
curses would actually come down on their
heads or not I dont know but the result was
they sent after us & told us we might go in
without any backsheesh. In the mosque the
tablets with Arabic writing on them excited
the particular admiration of my friends
& they declared that the art of Arabic chi-
rography was on a level with painting & that
such things were to be compared with the pictures
of Raphael. There was one tablet which
looked something like this
only more regular which
I should have supposed to
be a mere ornament, but
they read it. It was not this sort however which
they admired so much but another.
Constantinople is by all odds the most beau-

Peirce, Charles S., *Charles Sanders Peirce papers*, 1787-1951. 1870 September 2. Houghton Library, Harvard University, Cambridge, Mass. MS *Am* 1632 (L 341), p. 6. Image in Public Domain.

Image 33. [Contour of Peirce's right hand with flea bites. Letter of September 22nd, 1870]



Peirce, Charles S., *Charles Sanders Peirce papers*, 1787-1951. 1870 September 22, Syracuse, Italy. Houghton Library, Harvard University, Cambridge, Mass. MS *Am* 1632 (L 337), (seq. 24) p.10. Retrieved from https://nrs.harvard.edu/urn-3:FHCL.HOUGH:4927598?n=24. Accessed 11 Nov. 2021.

Image 34. Cerebro: a fictional artifact in the X-Men comics and films, used to visualize the location and calculate the movement of things in space and time.

Image 35. The *Liber Calculationis* is an extensive collection of astronomical and computistical texts, based on the Aachen Encyclopedia created a decade before. The *rotae* of the *Cursus Solis per Signa* diagram, drawn on folio 136, is divided in 24 parts (two per month) indicating the dates in which each zodiac enters and/or leaves each month.

Liber Calculationis, 818 AD. München, Bayerische Staatsbibliothek, BSB Clm 210, f. 136. Retrieved from https://www.digitale-sammlungen.de/en/details/bsb00047183. Accessed 10 Dec, 2021.
Image in Public Domain.

Image 36. Fra Angelico's fresco "Annunciation", c. 1440. Dimensions: 176 x 148 cm

Fra Angelico. *Annunciation*. 1440-1442. Basilica di San Marco, Florence, Italy. Retrieved from https://www.wikiart.org/en/fra-angelico/annunciation-1442. Accessed 06 Jan. 2022. Image in Public Domain.

Image 37. The image bellow collects about 7,500 images resulting from a search for “Artificial Intelligence” in Shutterstock.

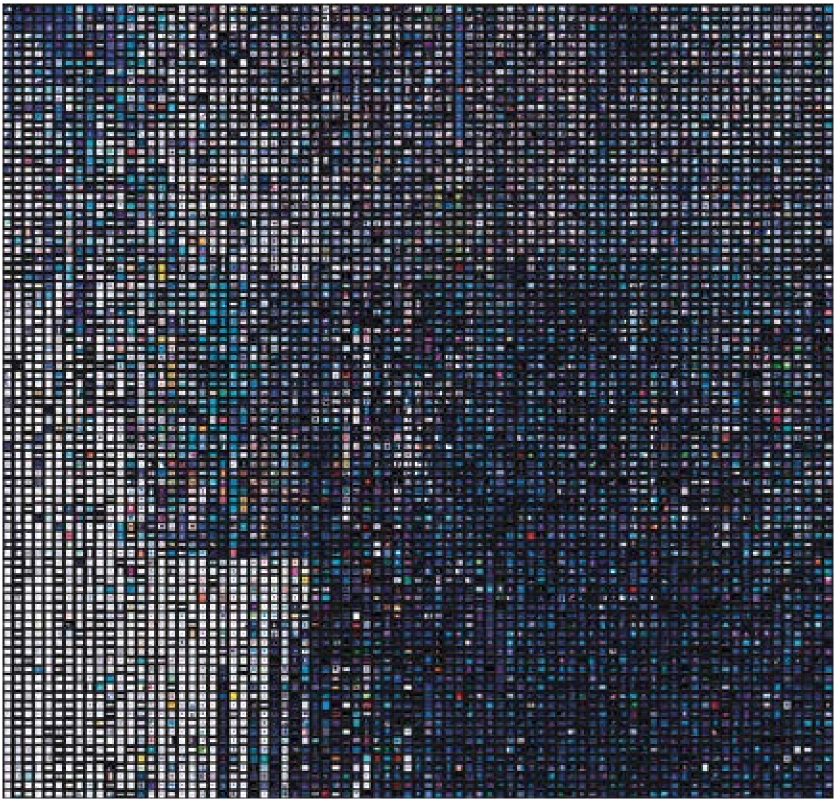


Image created by Dario Rodighiero.

Image 38. The *Bibles moralisées* were created to interpret and explain scripture. Selecting passages from the Biblical text, the moralized Bible shows in this illustration God's figure bent while in the work of creation, shaping the universe with the aid of a compass in his right hand, drawing and defining it.

Bible Moralisée, 1220-1230. Vienna, Österreichische Nationalbibliothek, Codex Vindobonensis 2554, f. Iv. Retrieved from https://commons.wikimedia.org/wiki/File:God_the_Geometer,_Codex_Vindobonensis_2554,_circa_1220-1230_(29304249070).jpg (photo author: Levan Ramishvili) Accessed 10 Dec. 2021. Image in Public Domain.

Image 39. Oil on canvas, painted by John Waterhouse in the beginning of the 20^{th} century. It illustrates Echo and Narcissus, from the *Metamorphoses* by the poet Ovid (43 BC – 17/18 AD).

John William Waterhouse, *Echo and Narcissus*, 1903, Walker Art Gallery: WAG 2967. Retrieved from https://artuk.org/discover/artworks/echo-and-narcissus-127992 Accessed 10 Dec. 2021.

Image 40. Clearly inspired by the myth of Narcissus, this image is part of a popular French medieval romance about the art of love. The *Roman de la Rose* is attributed to Guillaume de Lorris and Jean de Meung and was originally written in mid 13th century.

Roman de la Rose, 14th century, Llyfrgell Genedlaethol Cymru – The National Library of Wales, NLW MS 5016D, f. 11v. Retrieved from https://www.library.wales/discover/digital-gallery/manuscripts/the-middle-ages/roman-de-la-rose. Accessed 9 Dec. 2021. Image in Public Domain.

Chapters

1. Ramon Llull's *Figura Elementalis*. The Evolution of Diagrams and their Gnoseological Functionality

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Germany

Introduction

When Llull is discussed, everyone associates his name with combination mathematics, and he is remembered as one of the forerunners of computer language (Fidora & Sierra, 2011). This is because he draws up a logical and combinatorial system, with which he discusses the theological and philosophical truths of his time (Künzel & Cornelius, 1991). He himself describes this system as an "art." Llull's art was studied, in the 17th century, by Leibniz, who later adopted Llull's idea of universal language for his theories in the field of combination mathematics and logic (Colomer, 1979).

Llull's art appears as a tool for researching all sciences, becoming the science of sciences, a vision and an intuition (Domínguez, 2018), or, to use an expression that first appeared after Llull, a scientific method (Sales, 1998). By means of the correspondence between letters and principles, the Artist obtains combinations that reveal themselves to be nothing short of logical propositions concerning God, the world, dogmas, and articles of faith. The principles of the art are common and constant principles that can be shared by the three major current monotheistic religions. In other words, they are principles with claims to universality, which the interlocutor can accept as a basis for logical, and thus religious and philosophical, dialogue. Llull understands that the premises for logical and rational debate must be the same, and agreed upon, i.e., universal, otherwise no accord on the processes of discourse, and on their results, can subsist (Romano, 2002).

Without going into the heart of the workings of the art (Rubio, 2008), which would require a separate discussion, an important consideration is needed at the outset to clarify the most significant stages in the development of Llull's philosophy. The system of the ars experienced an evolution over time; this evolution led two academics, Lola Badia and Antony Bonner (Badia & Bonner, 1983), to identify four key stages, subsequently recognized by all Llull's scholars. The four phases are marked by the appearance of central works: 1. *Lògica del Gatzell – Liber contemplationis* (1271-1274). 2. *Ars compendiosa inveniendi veritatem* (1274) – *Ars demonstrativa* (1283)3. *Ars inventiva veritatis* (1290). 4. *Ars generalis ultima* (1307). The two middle phases constitute the commonly called quaternary phase and the ternary phase of the art. The quaternary art is so called because its center of inspiration lies in the theory of the four elements: the research into nature, and the study of God as prime cause, constitute the speculative cornerstones of this phase. Meanwhile, the ternary phase is informed by the reflection that the author cultivates regarding the theory of correlatives that constitute the threefold structure of every being. This stage reflects the maturity of the theological discourse on the trinity, and on how this can be discussed by the art.

The art makes use of definitions, rules, alphabets, tables and illustrations. The alphabets, the tables and the diagrams exploit their potential to present information visually, acting as aids to memory with an epistemological function.

The *Ars brevis* (1308) contains, for example, the following alphabet:

> B significat bonitatem, differentiam, utrum, Deum, iustitiam et avaratiam.
> C significat magnitudinem, concordantiam, quid, angelum, prudentiam et gulam.
> D significat aeternitatem siue durationem, contrarietatem, de quo, caelum, fortitudinem et luxuriam.
> E significat potestatem, principium, quare, hominem, temperantiam et superbiam.
> F significat sapientiam, medium, quantum, imaginatiuam, fidem et accidiam.
> G significat uoluntatem, finem, quale, sensitiuam, spem et inuidiam.
> H significat uirtutem, maioritatem, quando, uegetativam, caritatem et iram.
> I significat ueritatem, aequalitatem, ubi, elementatiuam, patientiam et mendacium.
> K significat gloriam, minoritatem, quo modo et cum quo, instrumentatiuam, pietatem et inconstantiam.

Corresponding to each letter is a set of principles. Corresponding to the letter B are goodness, difference, whether, God, justice, avarice; corresponding to letter C are greatness, concordance, what, angel, prudence, lust and so on. Also associated with the alphabet is the *tabula*, showing in visual form all the possible combinations between the letters of the alphabet. For example, the combination BCtB, applied to God as subject, could formulate the question: "Vtrum Deus habeat differentiam in bonitate et magnitudine?", namely whether there is a difference in God between goodness and magnitude.

The most important illustrations in the art are Figure A, Figure T, and Figure S. Figure A is the figure showing divine attributes or principles. Each principle is assigned a letter; B in the case of goodness, C in the case of greatness, F in the case of eternity, and so on. Figure S is the figure showing the rational soul, in which the combinations between its three powers (memory, intellect and will) are represented. Figure T is the figure of meanings, the *figura significationum*. This is very important, because it groups together the relative principles of the art, namely those concepts that regulate relations between subjects, and between all existing things. All three figures would undergo simplifications from the quaternary phase to the ternary phase. For example, Figure A would no longer have sixteen dignities, or principles, but nine, and Figure T would also see a reduction in the number of its triangles, from five to three.

The three figures are not independent of each other, their meanings are applied to the terms of the other figures, in line with the rules of artistic discourse. Llull writes in the *Ars compendiosa inveniendi veritatem* (p. 3):

> Nota, quòd, sicut S. est Figura communis in hac arte, sic similiter T. est Figura communis eidem arti; quoniam S. non potest inquirere nec tractare de arte sinè T., nec T. potest significare de A. V. X. Y. Z. sinè S.: quare ista ars praecipit, quòd homo sciat concordare S. T. in significando et inquirendo A. V. X. Y. Z.

The figura elementalis

Here we will focus on the elemental figure (Dambergs, 2000), its visual representation, and its gnoseological functionality – **Image 9**.

It may be strenuously asserted that the author shows a real, genuine interest in the philosophy of nature, speculation on which, in his day, could not be separated from a consideration of the dynamics of the four elements, regarded as the constituent principles of living material bodies. The four elements and their processes of mixture are, in medieval natural philosophy, the basis of all natural processes in the physical world. Knowledge of them is useful not only to medical science, in which Llull shows a certain interest (Pereira, 1979), but also to all of the philosophy of nature, to astronomy, and to cosmology. Indeed, the four elements refer to the dynamics of the astral influences on the life of men, and of all creatures, since stars and planets are also characterized by elemental complexions (Pring-Mill, 2007).

Llull assigns the first letters of the alphabet to the four elements, to be able to express natural phenomena with combination formulas (AB, CD, etc.), from which one sees the connection between natural philosophy and Llull's art.

The *figura elementalis* already appears in the at the beginning of the quaternary phase, namely in 1274 in the *Ars compendiosa inveniendi veritatem*. It is described as part of Figure T:

> Figura ipsius T est duplex: una circularis composita de dictis triangulis; alia de quattuor figuris elementalibus. (p. 42)

This statement is highly important, because it allows us to place the elemental figure within the compilation of the other logical figures of the *ars*. It does not put itself forward as a means of research that only relates to natural phenomena; it rather takes the form of a heuristic tool in general, because the operations of the four elements, namely the operations of nature, reflect the operations of logical and artistic discourse in the other areas of interest of the real:

> Figura elementalis significat in hac Arte operationem ipsorum A T V X Y Z; quia, sicut elementa operantur naturaliter, ita S. per simile in illis accipit demonstrationem in figuris istius Artis, loquendo metaphoricè cum illo, qui objicit, ut in illo simili convenienter possim habere meum propositum de ipso in caeteris figuris. (p. 42)

The elemental figure in the art means the operations of figures A T V X Y Z. From the operations of the four elements, the rational soul derives, by similarity, the demonstrations of the other figures, and founds logical discourse on the elemental metaphor.

In the *Ars compendiosa invenienedi veritatem*, Llull dwells on the visual representation of the four degrees of the elements in the elemental figure. In nature, the elements are four in number, and are found in living beings in differing amounts, which can be perceived thanks to experience, and to the bodily senses, and which can be measured in degrees. No element ever presents itself in a pure way; instead, each of them has a quality of its own, and one that is appropriated: fire is hot, and takes its dryness from the ground, air is moist, and draws its heat from fire, water is cold, and takes its moistness from air, while the earth is dry, and it derives its coldness from water. Hot, dry, cold and moist are thus the four elemental qualities present in the elements, that are always forming mixtures between themselves.

In visual terms, the four elements are identified by different colors. Fire is red, air is blue, water is green, and the earth is dark. Associated with each horizontal line of the quadrangle is one of the four degrees of the corresponding element. The first line corresponds, in the figure of fire, to the fourth degree of heat, the second to the third degree, the third to the second, and the last to the first degree of heat. The same procedure holds for the other three elemental figures.

In the later work *Lectura artis compendiosae inveniendi veritatem*, Llull develops the description of the elemental figure, and clarifies the application of Figure T to the four elements. He also discusses the arrangement of the spheres of the elements within the four quadrangles. The spheres of the elements, in cosmological terms, are arranged in accordance with their lightness and heaviness. Equally, we can observe that, in the first vertical line to the left of the figure of fire, the lightest element, namely fire, is also the highest; going down, we find air, water, and earth. However, the four elements are never found in their state of simplicity in nature, they are always found mixed together, according to the processes of generation and corruption of substances. The other lines of the *figura elementalis* reflect none other than the *mixtiones* of the elements, which are depicted according to different orders, namely according to their rising and falling movements.

If we want to understand the application of Figure T on the *figura elementalis*, it will be enough to cite an example. Let us take the triangle of Difference-Concordance-Opposition of Figure T, and let us start from difference. The principle of difference relates to all four elements, given that each has a different quality of its own. For that matter, relations of concordance can be established, in pairs, for those elements that share one of the two qualities: for example, fire and air have heat in common, a quality that belongs to fire, and is appropriated in the case of air. However, there is also a relationship of opposition between them, given that the quality belonging to air, moistness, goes against the appropriated quality of fire, dryness.

The concept that the elemental figure is a part of Figure T is reaffirmed by Llull in the *Liber principiorum medicinae* (1275), in which the author sets out the principles of medicine, representing the four elements and their qualities in the figure of a tree.

In it, the roots are the four human humors (yellow bile, blood, phlegm, and black bile) corresponding to which are the four elements: yellow bile is associated with the element of fire, blood is associated with air, phlegm with water, and black bile with earth. On the branches that lead off from the left-hand side of the trunk, Llull locates the principles of ancient medicine (*res naturales, res non naturales, res contra naturam*). The branches leading off from the right-hand side of the trunk contain the principles of Figure T of Llull's art (*principium-medium-finis, differentia-concordantia-contrarietas, maioritas-aequalitas-minoritas*), namely the three triangles of Figure T according to which the mixture of the degrees of medicines will be discussed. In the other part of the trunk, Llull locates the four elemental qualities and their gradation in simple medicines, assigning letters to them. A is heat, E is plant in the fourth degree of heat, F is a plant in the third degree, G is a plant in the second degree, and H is one at the first degree. This also applies to the other qualities. Furthermore, Llull gives instructions on how to make use of the tree diagram, and how to reproduce it. He writes (Bonner & Soler, 2006):

> Quicumque autem hanc artem intelligere et scire desiderat, oportet ipsum scire perfecte alphabetum superius positum et ipsum in memoria semper retinere; et etiam oportet ipsum facere flores alios similes floribus dictae arboris fabricatos et mobiles, ut alios ipsorum florum cum aliis commiscere et formare demonstratiue possit, necnon ipsos augmentare et diminuire ac etiam adaequare ad declarandum, qualiter medicus suas debet componere medicinas (Liber principiorum medicinae, p. 445).

This passage shows us that the art must be understood not only intellectually, by anyone wanting to make use of it, but also learnt by heart with respect to certain sections of it. For example, the alphabets are to be retained in one's memory *(in memoria semper retinere*). Moreover, the diagrams are to be not only drawn, but created, so that their parts have movable form, to make the geometric figures turn, thereby obtaining the various compositions or combinations.

At the height of his quaternary phase, Llull composed the *Liber exponens figuram elementalem*, between 1285 and 1287, in which the elemental figure is analyzed from new points of view. The author adds certain considerations, compared to his previous works, which seem to reflect his particular devotion, in that period, to the study of material substance, and in general to the philosophy of nature. Indeed, it is no coincidence that the *Lectura artis demonstrativae sive Liber chaos* also dates to this period, in which the Majorcan philosopher concentrates on highly complex issues of natural philosophy. Llull writes:

> Elementa sunt quatuor principia [...]; ut autem de ipsis et de operationibus eorum in corporibus physicis habeamus notitiam, per quam possimus propriè vel etiam metaphoricè respondere ad factas Quaestiones, sunt figurata in hac praesenti Figura Demonstrativae Artis ipsa quatuor Elementa. (Liber exponens figuram elementalem, p. 1)

After discussing the degrees of the elements and the arrangement of the spheres, Llull dwells on the position of the elements within matter. He asserts that fire and earth, found in the corners of the quadrilateral, represent the extremity of matter in the fourth degree of heat; he calls them *extremitates substantiae*. At the same time, these two elements are also located within matter, *intra substantiam*, compared to air and water, because they have, in the same substance, the maximum intensity or virtue. However, in material substance we find all four elements, and thus also the relations of opposition between them. Thus, the four median diagonal lines connect the opposites, the simultaneous presence of which in matter appears possible thanks to the different gradation of their qualities.

In trying to get inside the force of Llull's imagination, perhaps what he had in mind can be represented in this way, by likening the figure of fire to a peppercorn which, according to Llull's natural philosophy, is found in the fourth degree of heat.

Overlooking any amusement that such an image may very well produce, it nevertheless gives an insight into the multifunctionality of Llull's elemental figure. On the one hand, it serves the purposes of calculation, and acts as an aid to memory. On the other hand, it serves the purposes of representation that, intuitively, makes it comparable to modern atomic models – so to speak. Whereas Thompson, at the start of the 20th century, shows in his atomic model the protons and electrons within an atom, Llull does the same thing in representing within a peppercorn its constituting elemental particles, seeking to give it an arrangement in accordance with a logic which he makes very certain he explicates.

The same attempt, namely the representation of the elemental particles within material substance, is also present in the *Liber de nova geometria* (1299), and thus at the height of the ternary phase.

Image 10

In this picture, Llull depicts a plant in the fourth degree of heat differently from the original elemental figure. Indeed, this figure is composed of triangles and rectangles forming a rhombus with two additional lateral triangular extensions. In this figure, the four degrees of fire are represented by the letter A, the three degrees of earth by the letter C, the two of air by the letter B, and the sole degree of water by the letter D.

If we try to close the figure, we shall see that the heart of the composition will be constituted by fire, and that the lighter elements will constitute the highest part of the solid; the base will be comprised of the two heaviest elements, and earth will have a greater extension, in view of its higher degree compared to water. The figure also harks back to the Platonic tetrahedron of fire, which the author refers to probably in a conscious way. In the *Liber de nova geometria* Llull uses new figures for the four elements, such as for example the figura de *mixtione elementorum*, trying out new representations which, however, no longer seem to appear in his later output.

Conclusions

In the ternary phase, and here I conclude, the elemental figure is thus still present. One need only recall the composition of the medical work *Liber de levitate et ponderositate elementorum* (1294), in which the issue of mixture of degrees is still dominant within Llull's literary output, and the *figura elementalis* is the central figure of the work. However, in this ternary phase, the author experiments with new geometric representations, as shown by the *Liber de nova geometria*. Finally, in the last phase, the post-Art phase – namely from the *Ars generalis ultima* onwards – Llull would no longer use the elemental figure, resorting instead, always, to similarities between the elemental processes and the relationship dynamics of the other beings to explicate the theory of correlatives in relation to each degree of existence.

2. The *Horologium* diagram: Visual Representations of Time

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Introduction

The study presented in this article introduces the first results of a larger research about *Horologium* diagrams: a list of monthly and hourly (*per horae aequales*) concordances of the length of the sun's shadow, measured in *pedes* (a sixth of the human body's height), over the course of the year, for time-keeping purposes; it can be divided into groups of similar visual dispositions, and according to the context of their production and transmission.[1]

Knowing time

It is hardly possible to pinpoint when Humans started registering the changing cycles of the stars and using those records for their own benefit,[2] but it was of the utmost importance to track years and seasons for hunter-gatherer societies and even more so in any agricultural society.

Tracking time involves looking at the sky, as well as down at one's feet. It starts by observing and registering the cycles of the Sun, the Moon, and the Constellations to reveal the changing of months and seasons (the macro-cosmos) and ends with the shadow of the human body (the micro-cosmos) on the ground, produced by sunlight during daytime, helpful in tracking the duration of the hours and moments of each day. For the wheels of nature to be perceived as predictable, no more is needed. However, as societies evolved and grew, accumulated cosmographical knowledge would be used with far more acuteness and parsimony. It became necessary to establish calendars, to keep track of agricultural productions, religious events, tax collecting; predictions and prognostics became essential for medical treatments and for warfare; in sum, to organize community life, and to control it. And no ancient knowledge was dismissed, it was rather questioned and improved upon.

Adapting time

Since the beginning of Christianity's history,[3] a good calendrical *calculus* meant that Easter could be celebrated on a particular and consensual date in all territories where Christians were settled. It should not be placed on the same date as the Jewish Easter Feast and it should be limited by the Spring Equinox and the Palilias (foundation of Rome), and that was a difficult task to complete, considering it had to fall on a specific day of the week: a Sunday. In trying to create a calendar that could harmonize all requirements, many approaches were experimented and put to the test. After the 3rd century, catholic churches agreed that Easter should be after *Luna* XIV (first full moon after the Spring equinox); and the Council of Nicaea, in 325, agreed to 21st of March as the Equinox date (previously it was set on the 25th). Many versions of the *calculus* were since designed and redesigned to political and theological requirements while trying to reach a harmony between the lunar (11 days shorter) and solar calendars.

A stable prediction standard for the celebration of Easter services, throughout eastern and western territories, was soon to start spreading in the form of *computus* – a set of rules and calculations based on astronomical observation, used to create calendrical tables and timelines. In the 5th century, Victorius of Aquitaine (fl. mid-5th cent.) computed a perpetual table – that was still in use in the north in the 8th century; shortly after, Diosysius Exiguus (c. 470 – c. 544) used calculations for 5 cycles of 19 years (the Metonic lunar cycle) with 29 days for even months and 30 days for uneven months. In 725, Bede the Venerable (672/3 – 26 May 735) made a perpetual table for 532 years and was responsible for a standardized manual for calendrical astronomy that would become the most copied of the Middle Ages. A century later, Helperic of Auxerre's (fl. 9th cent.) *computus* also became very popular.

Computus copies frequently contained an authorial commentary explaining why the adopted version (some of which, mentioned above) was the best. One of the most copied commentaries on Helperic's calculation was that of Abbo Floriacensis (945 – 1004); a strong reference for Benedictine schools, Abbo was part of a very active and long-lasting text transmission tradition. The *computus* became a standardized model for learning and consulting, and a tool for monastic everyday life (dependent on agriculture cycles as much as on a spiritualized view of the world). In the 12th century, the golden number was introduced for computistical calculations, and by the end of the Middle Ages the Dominical Letter was widely adopted and eventually replaced Bede's method. Later, in 1582, the Gregorian calendar (New Style) was set.

Registering and teaching time

The centers for *computus* production during the Carolingian and Post-Carolingian periods (roughly from the 8th to the 12th century) were mainly Benedictine and spanned from the south of Europe to the British Islands. Through them, a systematic transmission and copying of texts and diagrams was tenaciously kept. Traveling, borrowing, and copying texts (and particularly textbooks) was quite common for monks and abbots. In dedicated *scriptoria* or in monastic or cathedral Benedictine schools, classical works related to natural philosophy were often present – frequently in the form of commentaries, but there were also chronicles (biblical and historical) being made, correspondence being compiled, Bibles and liturgical works being commissioned, texts by the Church Fathers and other theological literature were recurrently feeding libraries; however, *computus* materials were ubiquitous. They used visual tools that were often both inspired by classical literature and accommodated to Christian visual traditions – which also left room for constant improvement, sometimes in the form of aesthetic style variations, others in the form of new designs, corrections, accommodations and innovations.

The making of *computus* was not a job of mere copying, it involved the tailoring and refashioning of contents. An example of this is manuscripts SJCollege MS 17 and Cotton MS Tiberius C I, both copies of the same *computus*, kept at the British Library. They contain excerpts and chapters taken from known works by Bede and Pseudo-Bede, Eugenius of Toledo, Isidorus, Macrobius, Pythagoras, Abbo of Fleury, Cicero, Hyginuys, Pliny the Elder, Martianus Cappella, Eratosthenes of Cyrene and many anonymous texts, diagrams, and tables too. It is a clear example of a textbook that took a considerable amount of planning and knowledge of the subject (theoretical and practical) and served as a reference. Its compiler was familiar, and had access to, a vast collection of historical as well as scientific manuscripts. And this was only possible because monasteries promoted well-furnished libraries, where students, scholars and scribes could, and were, flourishing. In this case, the original compilation (or edition) is attributed to Byrthfyrth of Ramsey, himself a student of Abbo.

On most occasions, cosmographical diagrams were inserted in manuscripts/codices as part of computistical texts, in addition to various tables and charts: identifying important dates of the religious calendar and historical facts, sometimes comprising up to six centuries of calculations, were generally followed by diagrams that either explained how those calculations were made, compiled information for quick consultation, or helped to visually remember concepts and operations.

These diagrams informed readers about the position of stellar objects, most importantly the Sun, the Moon, and the Constellations visible in the sky, at any given moment. Planetary movements (orbits) intersected the Sun's own path (the ecliptic) through the course of the year, and were thought to influence the human body and personality traits, disease treatment efficiency or even a man's destiny (*computus* were also used for prognostics, mainly because hospitals were frequently part of Benedictine monasteries, abbeys or cathedrals); solar and lunar eclipses were important for chronicle writing and decision making; and time reckoning was intimately related to the calculation of standardized solar-lunar cycles, marking the beginning, middle and end of key hours of the day, moments and seasons, and, of course, the date of Easter – needed by every monk to fulfil the Rule. Along with this information, often the compiler would add some mathematical content, such as numeral and measurement correspondences (Roman, Greek, Hebrew, and Arabic), geometry or algebra. The sheer presence of timekeeping devices/tools, in virtually every Benedictine monastery, shows timekeeping was indispensable. From meridians clocks, water clocks or clepsydras, to stone clocks, wax-clocks, solar clocks or mechanical clocks, to astrolabes, medieval societies relied on a variety of age-old (and also new) devices to tell the time, day and night.

Complementary knowledge on cosmology and astronomy was thus considered part of the program for a complete computistical expertise, set in place in monastic schools and rooted in the foundational works of Martianus Capella in the 4th century (along with Pliny the Elder, Calcidius and later Macrobius), but more consistently after Alcuin of York's cultural intervention in the 8th century.

Calculating time: the Horologium

Horologia can be considered a visual archetype on their own right, the starting point being the text on *De concordia mensium*, as presented by Isidorus in the beginning of the 7th century (the knowledge and use of this concordance is, however, far older). The concordance of months and its relationship to the reckoning of time was first passed on as a simple list of fixed data concerning the sun's shadow length in *pedes* at different hours of every month – **Image 11**, it evolved to table-like visual arrangements, and from them into circular tables. These presented two key aspects of time keeping: solstices and equinoxes' cycles, and the length of the shadow cast by the sun during the day (taking the human body as a gnomon), for example: in March and October, at the first and last hour of sunlight a man's shadow is 25 *pedes* long.

These simplified formats kept being used alongside more elaborated diagrams, which later became standardized models. The intermediate design stages of the *Horologium* are clearly an upgrade, providing visual arrangements that sustain the existence of confrontation (in concordance) and circularity (in symmetry) in the information presented to the user/reader – **Image 12**. The choice to move from a circular table-like arrangement to wheels (divided in 12 months), to branching semi-circles (also divided in 12 months), was a movement towards a direct relationship to an object: solar clocks, with angled lines emanating from a central device, a *gnomon*.

They were commonly also entitled *Horologium Viatorum*, and their relationship to travelers (mainly Benedictines) is obvious. These diagrams were not to capture light but served as a reference to tell the time after their users had measured their own shadow, becoming a portable asset (even if just in one's visual memory).

At a certain point, in the second quarter of the 9th century, at Notre-Dame-de-Laon, the manuscript 422 (now at the Bibliothèque municipale Suzanne Martinet de Laon) is created. It contains an array of multicolored and geometrically intricate diagrams illustrating Isidorus' texts. On folio 53r there is an early version (the earliest to my knowledge) of a popular *Horologium* format – **Image 13** – that got extensively repeated in *computus* produced at the Abbey of Fleury (and elsewhere) for at least three hundred more years. Taking three important data: the ages of the Moon, the Zodiac and the sun, and the length of the shadow of the Sun, these diagrams presented a trinity of time-reckoning knowledge.

The most frequent three-part design is arranged like this: two smaller circles, placed side by side on top, are arranged as wheel diagrams (meant to be read inside-out in a clockwise motion): the *sphere lunae* presents the ages of the Moon and how much it shines every night throughout the lunar month. By observing how long the Moon shines in one day, the user could "place himself" on a calendar; the *sphere solis* combines the course of the sun throughout the Zodiac and how many hours of sunlight can be observed each month. One larger circle is placed bellow: the *Horologium Viatorum* is centered in relation to the upper diagrams. The information contained in it occupies only the lower half of the circle, divided in 12 months and with layers for the concordant hours. The upper part of the large circle is in part behind the smaller circles, making it a tight group: the three are visually attached, as if they were part of the same mechanims (where on manuscript 422 the *horologium* is smaller and only a semi-circle, leaning against the upper two).

The verbal content of the three-part *horologium* diagram trio itself is associated to Bede: in *De Temporibus* (written in the early 8th century), Bede addresses the concept of the arbitrary and conventional aspect of time units and provides a normalization for the division of the solar hour: 4 points = 10 minutes = 15 parts = 40 moments (1 moment = +/- 90 seconds of our time). The division of the hour into small units allowed for a thinner prediction in calendric tables, and to a better calculation of the ages of the moon in time-keeping diagrams. In chapter XVII *De lunae cursu per signa* from *De ratione temporum* (725) Bede describes the information present on the moon diagram, and introduces yet another arbitrary use of time measuring, the five-point hour. In some manuscripts, this diagram illustrates chapter XXIV *Quot horis luna luceat*, such as in SJCollege, MS 17, f.81v.

In the case of the sun diagram, it is harder to tell, as it appears linked both to Bede and to Pseudo-Bede and latter to Abbo of Fleury; sometimes it is associated to poems, such as those of Pacifico di Verona and Wandalbert von Prüm (both from the 9th century). As to the verbal data on the actual *Horologium Viatorum*, it refers to a table attributed to Paladio in his *Opus Agriculturae* (4th century), linking the concordance of the months to the measurements taken from the length of a man (the gnomon)'s shadow, in *pedes*. Bede presented the same table of month/shadow concordance with small variations, and his is the most frequent version in medieval manuscripts. As to the text that accompanies the diagram (sometimes right next to it, other a few folios apart), there are many versions, as mentioned above, authorial and anonymous, from the most generic to the most concrete. On two examples[4] of these three-part *horologium* diagrams, the text was originally clearly created and edited for teaching purposes, it clearly describes the three parts of the diagram, and not just data; including color coding and the implicit rationale for each instruction. An extract reads:

> In fact, the lower wheel, which is also the largest, is called *horologium*, revealing in each month with how many feet the shadow of the human body determines each hour of the day; and because it is known that the months always agree in this fact two by two, the twelve months are divided, registering six on one side, and the same number of months on the other side, and a remaining space between one and the other division. And agreeing on the number of hours, they must be entered on opposite sides, two by two, by which (feet) each hour of the day is determined; and in all of them the number of feet has been written down, and, because [the months] agree, so they are expressed by using the same colour.[5]

Notes

1 The research was conducted for the Project: "From Data to Wisdom. Philosophizing Data Visualizations in the Middle Ages and Early Modernity (13th–17th Century)", POCI-01-0145-FEDER-029717 – Fundação para a Ciência e a Tecnologia – Portugal 2020, 1st July 2018 – 30th June 2021. For more on the subject of time and *Horologium* diagrams, see: Morini, C. (1999). *Horologium e Daegmael nei Manoscriti Anglosassoni del Computo*. Vita e Pensiero – Publicazioni dell'Università Cattolica del Sacro Cuore; Wallis, F. (translator) (1999). *Bede. The Reckoning of Time*. Liverpool University Press; Holford-Strevens, L. (2008). *Pequena Histótia do Tempo*. Tinta da China; Severino, N. (2011). *Storia della Gnomonica. La storia degli orology solare dall'Antichita' alla Rinascenza*. Roccasecca; Pedro J. Novella, "*Relojes de Sol Canónicos*" V numeración, V.20, (self-edition), accessed 26 November, retrieved from https://ccs.saf-astronomie.fr/wp-content/uploads/2021/Info_Mail/IM_24_2021_PJ5.pdf.

2 The oldest stone carvings of early humans' moon-tracking records (the "Aurignacian Lunar Calendar") were found in France and Germany in the 1970's and date back to 32.000 BC. Source: https://sservi.nasa.gov/articles/oldest-lunar-calendars/

3 In this study, we cannot go further than stressing that Emperor Constantine (272 – 337) was responsible for the religious syncretism that resulted in an institutionalized Christian calendar of Feasts. Nevertheless, this calendar only reached its final form and full geographical observance with the Carolingian reform, some five centuries later.

4 Bern, Brugerbibliothek, Cod. 250, f. 14v, c. 1000 (Fleury) and British Library, Sloane MS 263, f. 49r-49v, 11th century (Lyon).

5 *"Inferior sane rota, quae est et maior, horologium dicitur, ostendens per singulos menses, quot pedibus umbra corporis humani singulas determinat horas diei. Et quia bini semper menses in hac re concordare noscuntur, diuisi sunt menses .XII. et hinc .VI. illinc idem .VI. descripti, relictumque intra utramque divisionem spatium. Et sicut in horarum quantitate concordant, ita abtrinsecus contra semet habentur inserti, singulisque subiectus est numerus pedum, quo singulas determinant horas dierum; sicut concordant, ita pari colore illum habentes expressum."* Transcription, Latin revision, and Portuguese translation by Professor Manuel Ramos (Faculdade de Letras da Universidade do Porto). Translated to English from Portuguese by the author.

3. Where Mountains are red Leaves and Rivers are blue Stems: Tenth century Arabic *Imago Mundi* as floral motifs

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The Manuscript

In the Bodleian Libraries, University of Oxford in the United Kingdom, there is a curious manuscript dated to 1190–1210 Egypt, as detailed in the catalog entry, and which text is erroneously attributed to ‘Arabānī, ‘Abd al-Ghanī ibn Husām al-Dīn, (?- 1450).[1] According to the online catalog,[2] the manuscript, compiled in Egypt between AD 1020 and 1050, with the rhyming title “Kitāb Gharā’ib al-Funūn wa-Mulaḥ al-‘Uyūn” (كتاب غرائب الفنون وملح العيون), which I will literally translate here as “The Book of the Oddities of the Sciences and the Marvels for the Eyes”, is a rather incomplete late 12th or early 13th century copy also probably made in Egypt and drawn from an anonymous work largely dedicated to astrology and geography, two fairly common themes in the Arabic Medieval manuscript tradition.[3]

The manuscript contains a number of unique illustrations, a significant number of circular diagrams, some rare texts describing the positions of the stars in the heavens and several colored world maps.

Furthermore, the web catalog of the Bodleian library goes on to add the following information: Succinctly, the manuscript is divided into two books, one on celestial matters, one on the Earth. In terms of contents, it [the manuscript] has the extent of 48 folios. The size of the page is 324 x 245 mm. The size of the written area is 291 x 226 mm. Its layout: 27 lines per page whereas the frame is ruled text area.

The writing or hand are medium-large Naskh script in dense black ink and with headings in red. In terms of decoration, it contains 23 maps and diagrams. As for the binding, it is disbound, binding retained [a] 18th or 19th century Ottoman binding in poor condition.

In 2014 however, the renowned editor Brill published a diplomatic edition of the Arabic text of this manuscript with an English translation prepared by Yossef Rapoport and Emilie Savage-Smith under the title "An Eleventh-century Egyptian Guide to the Universe: The Book of Curiosities."[4] The 800 pages study that includes a partial facsimile of the manuscript aim to establish the Arabic text divided in two books, book one with the title: on the Heavens and book two: on the Earth, in addition to an English-Arabic glossary of star names and four extensive indexes namely on 1) animals and plants, 2) astronomical and astrological terms, 3) peoples and tribes and last on 4) place names.

Hence, and in the paragraphs that follow, I will be mostly dealing with what I consider at the very least some urgent questions: Is the floral element exceeding its aesthetic significance by implying a rather abstract universal notion of a geographical space? What are the implications of botanical representations on the intellectual production environment and even scriptoria of Arabic medieval manuscript?

The scribal practices

Needless to say that one of the many benefits of the 2014 critical edition published by Brill is making the hard to access anonymous astrological manuscript readily available for scholars and readers interested in the world of the 11th century Arabic medieval manuscript practices.

However, and for the purpose of this study, I will turn to five consecutive folios containing five maps that I consider unique and very puzzling in nature. They represent the five world rivers known at the time: Nile, Euphrates, Tigris, Indus and Oxus.

Folio	River's name (English)	River's name (Arabic)
Fol.42a *part.damaged*	The Nile	صورة النيل
Fol.42b *part.damaged*	The Euphrates	الفرات
Fol.43a	The Tigris	دجلة
Fol.43b	The Indus	السند
Fol.44a	The Oxus	نهر جيجون

These five rivers that come in five consecutive folios are the only main rivers represented graphically in the manuscript in a peculiar way where each river is drawn in the center of a map of its own and hence their significance and importance. And whether these five main rivers represent the limits of the world known to the anonymous geographer, and the writer of the manuscript, remain an open question. However, one cannot but notice that the five representations of the five rivers are very akin to the form of flowers in their constitution: the mountains and spring sources are generally on top of the map drawn as red leaves and red stones and the downstream water tributaries and affluents coming out of the main stem are drawn as blue branches.

Interesting to note however, is that only the first damaged and thus incomplete map of the Nile river in fol.42a bears the title *Ṣūrat* (صورة), i.e., image or representation, which imparts a certain sense of completeness and exhaustive character to the map in question. Nevertheless, the second damaged representation, that of the Euphrates river, in addition to the three completely preserved maps, the Tigris, the Indus and the Oxus, nowadays known as Amu Darya, do not all bear the designation of *Ṣūrat* (صورة), and are rather referred to as they are in the written text simply as rivers. Therefore, it seems plausible to think that the subsequent omission of the word *Ṣūrat* (صورة) by the unknown copyist and the supposed Egyptian origin of the manuscript might indicate that the copyist only knew first-hand the representation of the Nile river and that he relied solely on textual descriptions when it came to the other four rivers.

For the sake of both rigor and brevity, I will describe in what follows, and in a condensed manner, the information contained in every one of the five folios and around the floral motives of both the complete and partially damaged diagrams.

a) In fol.42a – **Image 14**, the representation of the Nile river, unfortunately partially damaged, is the first representation that bears the resemblance of a floral motif. Thus, I will only describe what is still visible on fol.42a. although the caption accompanying the diagram on its right side was fortunately not lost and it could be eventually used to guess the missing text on the floral representation. The red petals represent a mountains ridge called "The mountain of the Moon" (جبل القمر), the green blue stem crosses four provinces (اقاليم), the province of the Equator (اقليم خط الاستواء), the province of the Muslims (اقليم المسلمين), the unspecified third and fourth provinces (الاقليم الثالث والاقليم الرابع) . However, the stem ends according to the written text in "Ocean sea" (البحر المحيط) probably the Mediterranean sea. According to the text written in the caption though, it has its astrological and pseudo-astronomical ascendants in Cancer (السرطان) in the Hour (الساعة) of Mars (المريخ).

b) In fol.42b, also partially damaged and lost, we read the name of the river Euphrates right on the top of the representation. It is a floral motif where the red petals are called the beginning of the Euphrates and not a mountain or a mountain ridge, as is the case with the Nile river. Its blueish stem receives countless small rivers, according to the text, and the stem ends finally in river Tigris. Also according to the text in the caption, its astrological ascendants are in the Spike (السنبلة), in the Hour (الساعة) of the Moon (القمر).

c) In fol.43a – **Image 15**, the diagram represents the river Tigris with less direct floral connotations. It starts on top with a nameless mountain only referred to on both sides as Mountain (جبل). Several cities like Mosul, Baghdad and Basra in nowadays Iraq are mentioned along the gray blue stem of the river that receives a significant number of other rivers. Last but not least, the Tigris astrological ascendants are the Lion (الاسد), in the Hour (الساعة) of the Sun (الشمس).

d) In fol.43b – **Image 16**, the representation of the river Indus is more similar in essence to the diagram of the river Euphrates. It is a clearly a floral representation that starts with red petals of an unidentified mountain and along the blue gray stem, the reader can see the names of India (الهند) China (الصين), cities in Bihar (بهار), and the river ends in the oddly spelled Akbet in Arabic for Tibet (اكبت). Also worth noting is the absence of any astrological description of ascendant(s), as in the case of the river Indus.

e) In fol.44a, the last river treated in this essay is the river Oxus in nowadays Tajikistan. It also starts in dark red nameless mountains and the blue green stem crosses several lands (بلاد الغرب الاولى) until it reaches a green sea, the Caspian sea (بحر قزوين) also joined there by another side or end river. The river Oxus has been assigned two ascendants, however, as the Sagittarius (القوس) in the Hour (ساعة) and Mercury (المشتري).

Arabic Medieval Cosmogony

Yet and away from the many philological and terminological considerations that this until recently almost unknown text is inevitably abound with, we should highlight the aesthetic significance behind the floral motif which goes beyond its simplistic shape to imply a rather abstract universal notion of a geographical space. This somehow novel botanical representation of geographic space has profound implications on the environment of intellectual production and, needless to say, on the general scriptoria of Arabic medieval manuscripts.

In her book "Medieval Islamic maps: an exploration,"[5] the author Karen Pinto speaks repeatedly of the intimate link between ṣūrat al-'ālam, arabic for Imago Mundi and the Indic concept of the world as a lotus flower with mount Meru in its center. Pinto brings the example of al-Biruni's ṣūrat al-'ālam, studied by Gerald R.Tibbetts, who alludes to this possibility of Indic influence when discussing the Persian kishwar system. To quote Tibbetts: "There is a possibility that the idea was ultimately derived from Babylonian sources, although there are resemblances to the Indian cosmographic concept of Mount Meru and the lotus petals."[6]

Arabic depictions of ṣūrat al-'ālam or *Imago Mundi* under the different forms of a flower motif and in many ways like its Indian, Persian and Babylonian counterparts often recur as a topoi in previous and later works by renowned Arab geographers and *Hommes de Lettres*. For instance, Arabic medieval cosmogony could be best illustrated by the image of creation as Hyacinth, an example we find in the encyclopedia under the title "Nihāyat al-Arab fī Funūn al-Adab" compiled by the 13th century polymath Shihāb al-Dīn al-Nuwayrī (1279–1333).[7] Speaking of the creation of the ocean by God almighty, al-Nuwayrī goes on to say the thirteenth-century "When God intended to create the water, he created a green hyacinth, fixed its length, breadth, and height. Then he regarded it with a majestic look; then it became a small quantity of water that was in constant motion. This apparent undulation and motion was only a trembling caused by the fear of God."[8]

These readings coincide *de facto* with a Foucauldian approach first exposed in Michel Foucault's book "Les Mots et les Choses" published first in 1966.[9] Foucault argues that one of the components of pre-17th century Episteme is representation, and this same representation is understood as a localized projection and resemblance of common worldly signs or symbols used to understand the world. It is nonetheless a projection that is intimately and directly connected to the signs or symbols it uses. Foucault proposes the simile of the brain and the walnut[10] where both elements share a big resemblance in representation and where we could clearly see that resemblance is a key to knowledge in both directions, etc.

I argue here that the representation of the Nile river in "the book of Oddities" is another complex example of the 'representation through resemblance' paradigm. The Nile river was often depicted in a similar fashion in other cases of ṣūrat al-'ālam or Imago Mundi from similar time frames, namely in the works of al-Idrissi and Ibn Ḥawqal.[11] However, in "the book of Oddities," the Nile river seems to be depicted on purpose as an independent entity in the natural world, a being or entity with botanical and, one could claim, even anthropomorphic forms, attributes and not to forget precise astrological ascendants.

In a sense, all the five rivers in "the book of Oddities" looked the same, they share the same feature of a flower motif and they represent almost a certain prototype of a botanical object that has red petals, a stem and end roots ending in some form of a container. They resemble a living entity, or a being in the word. In this manner, the unknown author or copyist decided that they could be easily understood and doubtlessly equally represented for posterity.

Notes

1 Here my transliteration of Arabic names and terms follows the *International Journal of Middle East Studies* (IJMES) style guide.
2 Online entry in the Bodleian library catalog retrieved from https://digital.bodleian.ox.ac.uk/objects/748a9d50-5a3a-440e-ab9d-567dd68b6abb/surfaces/9939b9f2-899b-49be-9d80-7b7b78f0e37b/ (accessed 6 December 2021).
3 Gacek, A. (2012). *Arabic manuscripts: a vademecum for readers*, Leiden;Boston, Brill.
4 Rapoport, Y, and Savage-Smith, E. (2014) *An Eleventh-century Egyptian Guide to the Universe: The Book of Curiosities*. Islamic Philosophy, Theology, and Science; 87. Leiden, Brill.
5 Pinto, K. (2017). *Medieval Islamic maps: an exploration*. Chicago: The University of Chicago Press.
6 Tibbetts, "Beginnings" 93– 94. Quoted in Pinto. Ibid. p. 389.
7 For a selected English translation of the 31 volumes of Nihāyat al-Arab fī Funūn al-Adab, please see Muhanna, E. (2016). *The Ultimate Ambition in the Arts of Erudition: a compendium of knowledge from the classical Islamic world*. New York: Penguin.
8 Ibid. Pinto. p. 162.
9 Foucault, M. (2019). *Les mots et les choses : une archéologie des sciences humaines*. Paris, Gallimard. The title is translated to English as "The Order of Things."
10 Ibid. Foucault. p.29.
11 Ibn Hawqal. (1964). *Configuration de la terre: Kitab Surat al-Ard*. Paris, Maisonneuve et Larose.

4. The Confluences of Science, Philosophy, and Art: Ibn al-Haytham's *Optics* and the European *Perspectiva* Traditions

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Ibn al-Haytham's Theory of Visual Perception

To research the premodern use of visual materials in elucidating the investigations of science, and their underpinning narratives, this study focuses on the legacy in optics of the 11th century Arab polymath al-Ḥasan Ibn al-Haytham (known in Latin as 'Alhazen,' or 'Alhasen;' born in Abbasid/Buyid Basra ca. 965 CE, and died in Fatimid Cairo ca. 1041 CE). This line of inquiry will also address the wider context of the adaptive assimilation of Ibn al-Haytham's theories of vision and light within the European *perspectiva* traditions in science and the architectural visual arts, from the high mediaeval period to the early-modern epoch.

The optical studies of Ibn al-Haytham were mainly embodied in his monumental *Kitāb al-manāẓir* ('The Book of Optics').[1] This opus was divided into seven books qua volumes as follows: Books I–II dealt with direct vision, light, and color; Book III examined the errors in direct vision; Books IV–VI focused on catoptrics by studying the reflection of light and its instruments as mirrors; Book VII treated dioptrics by investigating the refraction of light and its instruments as lenses.

Ibn al-Haytham's theory of visual perception, in Books I–III of his *Optics*, rested on his research on the nature and propagation of physical light, as well as the conditions of its rays' intromission within the eyes. He explained this with a series of geometric modelling diagrams that guided his experimental procedures. His theory of vision also included an examination of the anatomy of the eyes, the physiology of the optic nerves and their connections to the anterior part of the brain, while also giving a psychological analysis of the cognitive processes underpinning vision. Furthermore, he placed an emphasis on the situated embodied psychosomatic experiences of the observer as a cognitive agent who is always immersed within the ambient scene by way of ocular-motor kinesthetic bodily acts.[2]

Ibn al-Haytham studied the behavior of the physical rays of light (*ḍaw'*) in the manner they irradiate spherically from every point on the lit surfaces of the objects of vision, and how they propagate rectilinearly within the intermediate isotropic, continuous, and homogeneous transparent medium (*shafīf*), such as air, which exists between the eyes of the observers and the lit visible things that face them. In order to study these physical phenomena, he used an isomorphic mathematizing of natural philosophy (*qua physics*) as mediated via hypothetical-deductive observational experimentations (*iʿtibār*) under controlled testing and verification. This penchant allowed him to overcome the shortcomings in the classical theories of his predecessors, whether they advocated the Euclidean-Ptolemaic emission doctrine of the mathematicians (*aṣḥāb al-taʿālīm*), or the Aristotelian intromission thesis of the physicists (*al-ṭabīʿiyyūn*).

Ibn al-Haytham analyzed how the physical light-rays that fell orthogonally on the eye would pass rectilinearly across the pupil, cornea, and uvea's transparent membranes until they refract at the outer surface of the crystalline-lens (*jalīdiyya*) and then bend again as they pass into the vitreous humor (*zujājiyya*), which itself then touches the filaments of the optic nerve (*'asb al-'ayn*) at the skull-cavity of the eye socket. The crystalline-lens and the vitreous humor are not merely refractive organs but sensing ones as well. The light-rays cause physical sensations on the crystalline in an ordered configuration that corresponds point-by-point with the contours and details of the visible lit surfaces from which these rays irradiate rectilinearly. These ocular sensoria differ in their sensed intensities in proportion to the degrees of illumination on each emission point from the visible lit surface of the object of vision. The seen image has as such a tactile aspect to it in the sense that it is a sensory arrangement of how the physical light-rays touch the crystalline with varying subtle fiery intensities. The array of sensations in the crystalline are then transmitted via the vitreous humor to the filaments of the optic nerves of the left and right eyes, and these get united in the common optical nerve (*'aṣaba mushtaraka*), which itself splits again into two nerve-conduits that reunite in a single sensing organ in the frontal cerebral cortex (*muqaddam al-dimāgh*) that is the last sentient (*al-ḥāss al-akhīr*). Such detailed analysis was itself supported by the diagrammatic descriptions of the anatomy of the eyes and the optical nervous system that connects to the front of the brain, as shown in **Image 17**.

Having considered the geometrized physics of light-rays, and the conditions of their intromission into the eyes, and how this is explained through anatomical studies of the physiology of the eyes and the optical nervous system that connects to the last sensing organ at the front of the brain, Ibn al-Haytham moves on to a psychological analysis of the cognitive processes in visual perception and their situated embodied experiences. He offered a Gestalt theory of vision in his analysis of the workings of the mind in discernment (*tamyīz*), syllogistic comparative measure (*qiyās*), and inferential judgement (*ḥukm*), which amount to the recognition (*maʿrifa*) of what is seen, while also evoking in this context the interplay between memory (*dhikr*; *ḥāfiẓa*) and imagination (*takhayyul*; *khayāl*). This relates to the visual perception, via inferential judgement and recognition, of twenty-two intentional properties (*maʿānī mubṣara*) that cover the cognizance of the light and colors of the objects of vision, their distance, position, solidity, shape, size, separation from other things or continuity with them, the numbering of what is seen, its motion or rest, roughness or smoothness, transparency or opacity, shadows and darkness, beauty or ugliness, similarities or dissimilarities. All these visible intentional properties show that direct vision occurs by way of cognition, and not simply in terms of ocular sensations in physiological and neurological terms, and in articulating a philosophy of mind and cognitive psychology that is also grounded on a phenomenology of situated lived ocular-kinesthetic experiences in embodiment.

Ibn al-Haytham's *Optics* and the European *Perspectiva* Traditions

Ibn al-Haytham's *Kitāb al-manāẓir* offers us a concrete example in the history of ideas of textual transference across geographical, cultural, ethnic, religious, and historical boundaries. The Arabic text was translated into Latin in the 12th century in Toledo, possibly within the circle of the Italian scholar Gerard of Cremona (Gerardus Cremonensis; 1114–1187 CE). The manuscripts of the Latinate renditions of this opus were disseminated widely from Andalusia and Sicily across central and western Europe under the titles: *Perspectiva*, *De aspectibus*, and *Optica*. This adaptive transmission had an impact on the opticians at the Franciscan college in Oxford in the 13th century, as exemplified in the *Opus maius* ('Major Opus') of the Franciscan natural philosopher Roger Bacon (1219–1292 CE), especially in its 'Part V' that contained the *perspectiva* discussion of the physiology of sight, the anatomy of the eyes, the optic nerves and the brain, along with the physics of light-rays and their reflection and refraction. A detailed attentive commentary on Ibn al-Haytham's treatise was also composed by the Silesian Franciscan natural philosopher Vitello (Erazmus Ciolek Witelo; 1230–1280 CE). The Latin version of the *Optics* had also a direct impact in the 14th century on the German Dominican natural philosopher Theodoric of Freiberg (1250–1311 CE), particularly in his study on the rainbow. Furthermore, the Arabic *Kitāb al-manāẓir* received around that same period, in the early 14th century, an influential recension by Kamāl al-Dīn al-Fārisī (1267–1320 CE) in Iran, which carried the title: *Tanqīḥ al-manāẓir* ('Recension of the Optics').[3]

Ibn al-Haytham was an architect (*muhanddis*) as well as an optician, astronomer and geometer. The architectural sensibilities in his ***Optics*** resonated as such with the *perspectiva artificialis* of the Florentine Renaissance as embodied, for example, in the *Della Pittura* (*De Pictura*; 'On Painting') of Leone Battista Alberti (1404–1472 CE) and the '*Commentario terzo*' ('Third Commentary') of Lorenzo Ghiberti (1378–1455 CE).

The Latin translation of Ibn al-Haytham's *Optics* was eventually printed in 1572 CE in Basel, under the editorship of the German mathematician at the Collège Royale de France, Friedrich Risner (Fridericus Risnerus; 1533–1580 CE). This edition carried the title: *Opticae thesaurus. Alhazeni Arabis libri septem, nunc primùm editi, Eiusdem liber De Crepusculis et nubium ascensionibus, Item Vitellonis Thuringopoloni libri X*[4].This printed volume was widely circulated and consulted by René Descartes, Christiaan Huygens, and Johannes Kepler. The illustration in **Image 18** is the 'Frontispiece' from the Risner edition, which illustrates the optical phenomena of perspective in direct vision, the rainbow, reflections in mirrors, refractions in a pool of water (*Triplicis visus, directi, reflexi & refracti, de quo optica disputant, argumenta*). Moreover, this image conveys a visual historicized narrative around the Roman siege of the Hellenistic Syracuse (213–212 BCE), and the use of Archimedean burning conical mirrors to incinerate the ships of the invaders.

Ibn al-Haytham's legacy in the science of optics, with its geometrized physics and experimentations, further inspired the 17th century Belgian architect, optician, and geometer François d'Aguilón (1567–1617 CE) at the Jesuit mathematical school in Antwerp, who in turn influenced the projective geometry of the French architect Girard Desargues (1591–1661 CE). The impress of the Latin translation of Ibn al-Haytham's *Optics* on d'Aguilón was set in the latter's treatise that was published in print in Antwerp in 1613 CE under the title: *Opticorum libri sex: philosophiis juxtà ac mathematicis utiles* ('Six Books of Optics, useful for philosophers and mathematicians alike').[5] This text was illustrated by the Baroque Flemish painter Sir Peter Paul Rubens in a series of vignettes that were engraved by Théodore Galle. Ruben's vignette, which is shown in **Image 19**, appears on folio 356 of the *Opticorum* (*Liber quintus: De luminoso et opaco*). The image reconstructs one of the many experiments that were described in detail by Ibn al-Haytham. In this case, it shows how light-rays propagate rectilinearly across a homogeneous isotropic transparent medium, such as air, and that they do not change the directional straight trajectory of their irradiation, nor intermix when crossing each other. The fires in the two lamps irradiate spherically in every direction, and the light-rays emitted pass as such through the circular hole in the surface facing them, leaving a circular corresponding illuminated area within the obscured small room behind the hole. Each of the lit circles corresponds to straight-line isomorphic mappings of the connections to its correlative lamp via the contours of a single circular hole. The illustration also displays a bearded man who represents the experimenter/observer who is inspired by the angelic muses. A similar use of the experimenter and the angels appears in all the vignettes of Rubens in the *Opticorum* to illustrate the experimentations through artistic visual renderings, including anatomical illustrations.

The *Opticorum* belongs to a longstanding tradition in architectural thinking and praxis, whereby notable architects composed treatises that related to the liberal arts, as well as making contributions to the fields of optics and geometry. This goes back for example in antiquity to the *De architectura* of Marcus Vitruvius Polio (80–15 BCE), and it is evident in the works of Ibn al-Haytham, who started his career in Abbasid/Buyid Basra as an architect (*muhanddis*) and travelled to Fatimid Egypt to design and construct a water-barrier to control the ebb and flow of the Nile. Realizing that such project was unfeasible considering his age and its building techniques, he became concerned over his own safety, fearing punishment from the dissatisfied Fatimid Caliph, al-Ḥākim bi-Amr Allāh (985–1021 CE). He therefore feigns madness and remains confined in his apartment in the vicinity of the Azhar Mosque until the death of Caliph in 1021 CE. It is perhaps in this period that the experimentation and composition of his *Optics* took form. We herein witness again the case of an architect who is polymathic as a geometer, optician and astronomer, with an impact of his legacy on like-minded architects such as d'Aguilón and Desargues – **Image 19**.

The influence of Ibn al-Haytham's *Optics* and his studies on geometry and astronomy is also visible in the work of the Polish-German astronomer Johannes Hevelius (Johannis Hevelii; 1611–1687 CE). This is particularly the case with the latter's treatise: *Selenographia, sive Lunae descriptio* ('Selenography, or a Description of The Moon'), which was published in 1647 CE in Gdańsk in the Kingdom of Poland.[6] This text constituted a milestone on the study of the moon and its topographic mapping. Its 'Frontispiece', as shown in **Image 20**, depicts two foundational figures of this discipline, who are standing on pedestals. To the right-side, we see the astronomer and physicist Galileo di Vincenzo Bonaiuti de' Galilei (1564–1642 CE) standing on the pedestal of '*sensu*' ('sensing'), holding a telescope in highlighting his contributions to selenography by way of visual observation as aided by an optical instrument. On the left-side, we see an artist's depiction of Ibn al-Haytham (named in a variation of the Latinate rendering of his name as 'Alhasen' ['*Alhazen*']), and standing on the pedestal of '*ratione*' ('reasoning') while holding a compass and a geometric diagram, highlighting his mathematical grounding of selenography. The significance of this frontispiece is engaging from the perspective of the history of science in the 17th century, since it credits not only a contemporaneous figure from the European culture and its Christian milieu, such as Galileo, but highlights the indebtedness of selenography to the premodern contributions of Ibn al-Haytham, not only more than 600 years before in history, but by also crossing the cultural, ethnic, and religious boundaries in recognizing the impact in science of the transmitted legacy of a Muslim Arab polymath in Europe.

Notes

1 Ibn al-Haytham, *Kitāb al-manāẓir*, ed. A. I. Sabra, Books I–III (Kuwait, 1983) and Books IV–V (Kuwait, 2002); Ibn al-Haytham, *The Optics, Books I–III*, trans. A. I. Sabra (London, 1989); Alhazen, ed. Friedrich Risner (Basel, 1572), with the Latin critical edition and annotated English translation by A. M. Smith (Philadelphia, 2001–2010). The Arabic title, *Kitāb al-manāẓir*, is usually translated in English as the 'Book of Optics', however, it can also literally mean the 'Book of Perspectives', the 'Book of Visible Aspects', or the 'Book of Sceneries'.

2 This paper is partly based on my earlier publications on Ibn al-Haytham: Nader El-Bizri, 'Alhazen's Theory of Vision', *Micrologus* 29 (2021): 21–33; Nader El-Bizri, 'Arabic Classical Traditions in the History of the Exact Sciences: The Case of Ibn al-Haytham', *The European Physical Journal Plus* 133 (2018): 271–277; Nader El-Bizri, 'Desargues' oeuvres: On Perspective, Optics and Conics', in *Visioning Technologies in Architecture*, ed. G. Cairns (Aldershot: Ashgate, 2016), 36–51; Nader El-Bizri, 'Seeing Reality in Perspective', in the *Art of Science: From Perspective Drawing to Quantum Randomness*, eds. R. Lupacchini and A. Angelini (Dordrecht: Springer, 2014), 25–47; Nader El-Bizri, 'By Way of an Overture: Classical Optics and Renaissance Pictorial Arts', in *Recto-Verso: Redefining the Sketchbook*, eds. N. El-Bizri with A. Bartram and D. Gittens (Aldershot: Ashgate, 2014), 11–26; Nader El-Bizri, 'Classical Optics and the *Perspectiva* Traditions Leading to the Renaissance', in *Renaissance Theories of Vision,* eds. C. Carman and J. Hendrix (Aldershot: Ashgate, 2010), 11–30; Nader El-Bizri, 'Ibn al-Haytham et le problème de la couleur', *Oriens-Occidens* 7 (2009), 201–226; Nader El-Bizri, 'In Defence of the Sovereignty of Philosophy: al-Baghdādī's Critique of Ibn al-Haytham's Geometrisation of Place', *Arabic Sciences and Philosophy* 17 (2007), 57–80; Nader El-Bizri, 'A Philosophical Perspective on Alhazen's Optics', *Arabic Sciences and Philosophy* 15 (2005), 189–218; Nader El-Bizri, 'La perception de la profondeur: Alhazen, Berkeley et Merleau-Ponty', *Oriens-Occidens* 5 (2004), 171–184.

3 Kamāl al-Dīn al-Fārisī, MS Istanbul, Topkapı Kütüphanesi, Ahmet III 3340 (copied at Nīšāpūr, 15 Šaʿbān 716/1316); Edited in 2 volumes in Hyderabad (Deccan), 1928–1930.

4 Friedrich Risner, Ed., *Opticae thesaurus*, Basel, 1572; with introduction by David Lindberg, NY: Johnson Reprint, 1972. Basileae: Per Episcopios, 1572; Federici Risneri in *Alhazeni Arabis Opticae thesaurus. Alhazeni Arabis libri septem, nunc primùm editi, Eiusdem liber De Crepusculis et nubium ascensionibus, Item Vitellonis Thuringopoloni libri X.*

5 Francisci Aguilonii e societate Iesu; Antwerp: Jan Moretus Widow & Sons (Antverpiæ: Ex officina Plantiniana, apud Viduam et filios J. Moreti, 1613).

6 Johannes Hevelius, *Selenographia, sive, Lunae descriptio* (Gedani: Autoris sumtibus, typis Hünefeldianis, 1647).

5. About minimal refractions: drop-colors and natural mirrors

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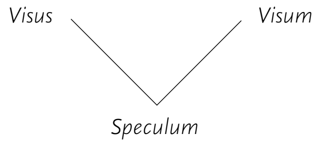
Introduction[1]

From Aristotle to Dietrich of Freiberg, the base of rainbow qualitative explanation is the relation between sunlight, water drop particles in which rays are reflected, and the observer's place. The diagrams from Dietrich's *De iride* (1304) represent the internal process of reflection and refraction in each droplet, but these schemes were not enough to prove how the incidence angle of the ray's light and the angle of its refraction configure the bow to the observer's perspective – **Image 23**.

Nonetheless, the similarity between the cosmological spheres' color order – in *Liber floridus* (1121) for instance (**Image 21 & 22**) – and medieval theories of rainbow chromatic disposition is rather meaningful. Thus, the theorical explanations relate to the observer's perception of light refraction, while spheres inspired the color structure regarding the elemental composition that flows from sunlight to earth throughout elemental spheres.

This issue was pointed out by the author of De radiis – attributed to Al-Kindi – who handled the natural light accidents and their influence on sublunar bodies under three different aspects: the distances between stellar bodies and objects, their elemental composition, and the variations of light incidence on them (*obliquitas aspectus*).[2] In a broad sense, the expression *obliquitas aspectus* represents the geometrical description of light refraction according to the medium density and the objects configuration. Despite the assumption that *De radiis* is a magical treatise, those aspects clearly overlap on some issues that are very known in optical texts. The experience of colored light phenomena joins the question of *obliquitas aspectus*. In the *Meteorologicum*, Aristotle described the rainbow as phenomenon which is simultaneously perceptual and physical. On the one hand, color depends on the observer's position, the physiological eye's aptitude, and, on the other, it depends on some specific meteorological conditions and the physical performance of stelar motions.

The author of *De radiis* was not far from Aristotle, and both from the medieval authors who dealt with light phenomena. The “science of aspects” (*De aspectibus*), according to Al-Farabi – in *De scientiis* – states that geometry has an “active” realm in which “lines and surfaces” would be studied on bodies, just as carpenters or smiths do, but from the perspective of visual and light phenomena. This consideration of embodied geometrical principles has in the science of appearances, *scientia aspectuum*, a discipline devoted to direct sight and the way in which light rays pass through different mediums – air, water – or fall over bodies becoming straight rays, reflexed, conversed, or refracted. Al-Farabi launched the first rule of refraction as the oblique lines that come out from the mirror according to this diagram[3]:



Reflection and refraction require homogenous and polished surfaces made with glass or iron, but it happens as well in wet fog, water, or other similar bodies. That diagram quoted literally in *De partibus philosophiae* by Dominicus Gudissalinus represents the emergence of this issue in the 12th century: the light’s incidence on a surface and the observer.

Robert Kilwardby, in the *Ortum scientiarum*, talks about the *modus demonstrandi* of the discipline that he calls *perspectiva*, for Al-Farabi *De aspectibus*, in which geometrical principles such as angle congruence are applied to light rays and their reflected forms on different mediums and surfaces. Quoting Aristotle, Robert asserts that geometry principles explain the causes of light accidents while rays are the objects of this discipline. In *perspectiva*, lines are physical phenomena, while in the rigorous sense geometry’s lines lack physical reference[4]. The difference between the line as a demonstrative principle and the embodied line as physical phenomena of light rays is missed in Al-Farabi *De scientiis* but acknowledged by Kilwardby. Robert Grosseteste does a similar exposition in his commentary on *Posterior Analytics*. He is more explicit in considering the incidence of the rays’ angles on mirror surfaces, attributing to those angles the proof of geometric disposition of light reflection:

> It is demonstrated in *perspectiva* that all two angles of which the incident ray represents one mirror, and the rest of the reflected ray are equal to the two radiant angles, and this conclusion is proven by this geometry. For proportions and the other types of angles, as they regard themselves, are equal.[5]

Grosseteste clearly points out the relation of proportionality, equality, and congruence between the angles of incidence and refraction on the mirror surface. Rays become the lines of triangles (*latera radiosa*) and the angles inscribed on those figures. Geometry allows the abstraction of lines and angles from the physical phenomena. This discipline establishes the general rules of demonstration about light phenomena demonstration.

Drop-colors and minimal refractions

In his treatise about color, Dietrich of Freiberg is unequivocal about the principles that explain our perception of color: the medium composition and the light ray's incidence. He points out the example of the light oblique incidences and its refractions that take place in a hexagon prism (*lapidus hexagonalis*) which is like a diaphanous mirror. This diaphaneity is precisely the "mirror nature" that some objects, like prisms or glass hexagons, share with water recipients or spider nets. Those objects become "natural mirrors" that spread colors because light flows along the points, lines, and surface of the bodies. Thus, the geometrical configuration of light and objects modifies their diaphaneity, a quality that shows their disposition to project light refraction. It is not a surprise that one of the main sources of Dietrich's works about color and light phenomena was book fourth of Ibn Haytham's *Perspectiva*. This book shows the different dispositions of mirror refraction on flat, convex, and concave surfaces. Specifically, Dietrich focusses on column mirrors. Those mirrors' surfaces show a configuration in which concave and convex light refractions could be experienced according to the incidence angles of light on exterior and interior surfaces. That sample of light incidence helps to establish the properties of the natural bodies' diaphaneity that represents their capacity of reflecting and multiplying light.[6]

In natural objects, mirror configurations indicate their capacity to receive light incidence according to three properties, which Dietrich calls "natural mirrors."[7] The first is the objects' natural shape that modifies the light incidence and the number of light refractions; the second is the place from where mirrors project light, since the surface's shape generates different intersections of light rays, as one can see in concave or flat mirrors; and the third is the object's density and its dimension, since the rays of solar light enter and exit through different densities that change their trajectories the same way as the internal incidence into the reflecting object. The difference between the objects' diaphaneity is an important issue in Ibn Haytham's work because he analyses natural bodies from the perspective of the different configuration of mirror surfaces, and Dietrich of Freiberg takes this interpretation in his hypothesis about the light internal refraction inside water drops to explain the refraction of the rainbow's colors.

Dietrich's obsession around hexagonal prisms justifies his approach to the light reflection inside the water drops. The well-known experiments that Dietrich, and other medieval masters, made with water glass recipients clearly show the light-colored decomposition. However, those experiments are not enough to explain the rainbow's color disposition and the colored curve in the sky. Dietrich overlays the properties of the hexagonal prisms on the water drops' spheric shape to explain the incidence of light from the air into the water drops. This minimal superposition of hexagons on many circles in each tiny water drop makes me think in a hexagonal prism inside any water drop to explain how the hexagonal light incidence explains the light refraction – **Image 23**. In *De iride*' graphics, this superposition of geometric figures and the simulation of natural experience in a geometric diagram becomes clear. Nonetheless, Dietrich's demonstration departs from the conception of objects as natural mirrors that accomplish in many ways the behavior of mirror surfaces in relations of light incidence, as shown by Ibn Haytham.

Personally, I think that the big issue in Dietrich's demonstration about minimal light refractions is his astonishing conception about objects as natural mirrors and the plausibility that almost everything in nature behaves like a mirror, according to the different samples of mirror surfaces considered by Ibn Haytham. In some sense, it is enough to find the compatible model of mirror surface in natural objects in which one can experience the light incidence and its refractions. Another aspect which is quite relevant in Dietrich's thought is the cosmological source of light. The stellar provenance of light rays is also highlighted by *De radiis* in which the disposition of elemental spheres contributes to the essential composition of sublunar objects according to the elements (fire, air, water, earth) and its qualities (hot, dry, moist, and cold). Light stellar way collects the elemental variations of their qualities and the visual representation of their influence in the light-colored refraction.

The issue of the rainbow's color order is partially resolved by the cosmological schemes of elemental spheres, since they represent the high order of color source. Sublunar objects represent in their composition the influence of light rays and how their incidence transforms them in refractive mirrors of cosmological colors – **Image 21 & 22**. Comparing the order of rainbow colors proposed by Dietrich of Freiberg and the cosmographical disposition of colors in the *Liber floridus* diagrams, it is remarkable how the medieval conception of the light ray's influence transforms the scientific experience about light incidence into a mathematical issue that depends on the careful survey of mirror surfaces. Dietrich was able to overlap a mirror – hexagonal prism? – in a water drop because he was convinced that natural mirrors spread their elemental composition through the light refraction in which colors emerge as elemental signs of cosmic spheres and the conception of natural mirrors.

Nature's Glassy Essence

[...] our Glassy Essence is something we share with the angels
Richard Rorty[8]

Last century, Richard Rorty stated that the representational conception of mind depends, in its broad sense, on the mind's mirror metaphor by which history of philosophy establishes a non-material background about mental activity. He calls "our glassy essence" the detached nature of mind conception that lacks corporeal or material aspects. Every mental, or soul, representational content is far from matter. Despite the quotations from ancient philosophy and some crude generalizations about medieval thinkers, Rorty neglected the material and geometrical history of "glassy essence." The medieval masters quoted above – who followed, through Ibn Haytham, the optical tradition – accomplished a complex task of establishing material and geometrical criteria about "our glassy essence" and nature's "glassy essence." Bacon, Grosseteste and Dietrich of Freiberg, as other medieval thinkers, had taken the material variables of knowledge into account and tried to set up an order among corporeal aspects of cognition trough geometrical principles.

For this task, Bacon gathered the soul's mirror metaphor, well known in Christian and Islamic traditions, in which light is the vehicle of knowledge. In parallel, he studied the sources of a discipline that is an application of geometry, described by Al-Farabi as the science of appearances (*De aspectibus*, afterwards *Perspectiva*) in which it is possible to know the set of situations in which the specific observer's visual perception and the mirror's imagery take place. The perceptual and material variables are the medium, position, physiological patterns, surface configurations and light incidences. All those aspects are described by geometrical principles. In this science, mirrors are material objects with specific characteristics; among them are geometrical structure and fine surface textures. My humble hypothesis is that the medieval masters proposed a geometrical solution for the mirror textures of "natural glassy essence."

According to Rorty, owing to the treatment of the mind's mirror metaphor, material and corporeal aspects of knowledge are not real philosophical questions. However, the tradition of *De aspectibus* shows an overwhelming interest in material variables of visual experience. *De radiis*, whose manuscript tradition reveals connections with mirror treatises (*De speculis*) ascribed to Euclid or Ptolemy, introduces a conception of rays of light whose shine passes through different mediums, from inorganic (*inanimatum*) to organic (*animatum*). The origin of the rays and its spreading is the sunlight and the refraction on celestial bodies that arrives to earth's convex surface. Furthermore, it is not a surprise that *De radiis*' author begins this work with an explanation about how universals are configured in the human intellect by the perceptual affections and the light's spreading:

> Therefore, it is proved by reason that the rays of all the stars have different operations in the things of the world according to the different properties of the same things, since all things originate and extent through their rays.[9]

Rays of light trespass all-natural bodies. Its properties and operations generate forms in matter that have specific time-space conditions. About knowledge, *De radiis* states that human as "minor mundus" gets the rays in his *spiritus ymaginarius*, in which rays print the images of singular things and their universal form. This spiritus is a "sacred mirror" where images achieve linguistic and conceptual signification. The rays' causal force is physical, the same as magnetism and mirror reflection.

A medieval master who knows Al-kindi's optical works possibly sees in *De radiis* an unfinished exposition which lacks geometrical and physiological background. However, *De radiis* roughly accomplishes the task that Roger Bacon or Dietrich of Freiberg proposed in their scientific work: demonstrating the way in which the light's rays spread through every medium, inorganic and organic, showing the perceptual structure of human knowledge in different ways. This structure is "our glassy essence" (the polished or porous soul-mirror) whose nature would be analyzed by the dispositions of straight or bend lines. Thus, medieval masters found a hypothesis about how "natural glassy essence" keeps a continuity between embodied and disembodied knowledge. Mirrors are simultaneously an artificial construction, an organic constitution of human perception and knowledge, a structure of light, motion, and change of natural phenomena.

Of course, I disagree with Richard Rorty, because "our glassy essence" as a metaphor and a conceptual structure is not entirely an incorporeal mind conception. Then, mind-mirror is not an angelical issue. Furthermore, we can recover a passage from one of the philosophers who uses the expression "glassy essence" – before Rorty – to connote the relation between intellect and extramental objects, Charles Sanders Peirce:

[...] all mind is directly or indirectly connected with all matter, and acts in a more or less regular way; so that all mind more or less partakes of the nature of matter. Hence, it would be a mistake to conceive of the psychical and the physical aspects of matter as two absolutely distinct aspects.[10]

Notes

1 Dietrich of Freiberg, *De iride et radialibus impressionibus*: Leipzig MS. 512, ff. 65v–66r (14th century). Lamber de Saint-Omer, *Liber Floridus*: BNF lat. 8865, f. 56v (c. 1250); Ghent MS. 92, f. 225v (13th century).

2 Al-Kindi (1974). De radiis. In M-Th. D'Alverny, F. Hudry (Eds.), *Archives d'histoire doctrinale et littéraire du Moyen Age 41*, p. 224.

3 Dominicus Gundissalinus (1903). De divisione philosophiae. In L. Baur (Ed.), *Beiträge zur Geschichte der Philosophie und Theologie des Mittelalters*, IV, 2-3, Münster, p. 113.

4 Robert Killwardby (1975). *De ortu scientiarum*. A. Judy (ed.). PIMS, Toronto, p. 81.

5 "Demonstratur in perspectiva quod omnes duo anguli quorum alterum constituit radius incidens cum speculo et reliquum radius reflexus sunt duo anguli radiosi aequales, et haec conclusio probatur per istam geometrie: omnium duorum triangulorum quorum unus angulus unius est aequalis uni angulo alterius et latera aequales angulos continentia sunt proportionalia, reliqui anguli prout se respiciunt sunt aequales", Robert Grosseteste (1981). *Commentarius in Posteriorum Analyticorum libros*. P. Rossi (Ed.). Firenze, Olschki, p. 149.

6 Smith, A. M. (2008). Alhacen on Image-Formation and Distortion in Mirrors: A Critical Edition of Book 6 of Alhacen's *De Aspectibus*. *Transactions of the American philosophical society, 98*(1), i-153, p. 39-48.

7 Dietrich of Freiberg (1985). De iride et de radialibus impressionibus. In R. Rehn, L. Sturlese, W. A. Wallace (Eds.), *Opera omnia IV. Schriften zur Naturwissenschaft*. Hamburg, Felix Meiner Verlag (Corpus Philosophorum Teutonicorum Medii Aevi, Band II, 4), p. 191.

8 Rorty, R. (2009). *Philosophy and the Mirror of Nature*. Princeton UP, Princeton, p. 43.

9 "Quocirca convincitur ratione quod omnium stellarum radii diversas habent in rebus mundi operationes secundum earundem rerum proprietates diversas, cum omnes res per radios oriantur et extent", Al Kindi, *De radiis*, op. cit., p. 220.

10 Peirce, C. S. (1992). Man's Glassy Essence. In N. Houser, Ch. Kloesel (Eds.), *The Essential Peirce I (1867–1893)*. Indiana UP, Bloomington, p. 349.

6. Representing Perfections, Latitudes, Qualities and Motions

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The Calculatores and the Background

The 14th century saw a considerable increase in the treatment of many physico-mathematical problems handed down in the tradition of the commentaries on Aristotle or in the commentaries on Peter Lombard's *Sentences*. From the third decade onwards, this trend was influenced by the works in logic, physics and mathematics of a group of authors known as the "Oxford calculators."[1] The most important among them are Walter Burley, Richard Kilvington, Thomas Bradwardine, William Heytesbury, John Dumbleton, Roger Swineshead and, above all, Richard Swineshead, the author of *Liber calculationum*, a work that became widely known in Italy from the second half of the 14th century. Calculators focused on problems directly or indirectly linked to the notions of infinity and continuity. They emphasized all those problems related to quantification or where a formulation and selection of the questions in terms of quantification was possible. Their approach, however, was not guided by an interest in mathematics alone, but also by logic and philosophy of language. Typically, they concentrated their efforts in composing special treatises rather than commenting on the Aristotelian corpus. Their main topics in the field of natural philosophy – often covered by single treatises – were *maxima et minima*, *incipit et desinit* (also *de primo et ultimo instanti*), the *actio* and above all the *reactio* of qualities, the *intensio et remissio formarum* and *de proportionibus motuum* or *velocitatum*. From all these "analytical languages" only this last one was almost exclusively dedicated to mathematics, or better, to the mathematical treatment of a problem belonging to natural philosophy: Which is the best way to express the relationship between forces and resistences causing the motion? Strongly connected to semantics and logic, the quantifying approach of the calculators was often proposed in collections of logical puzzles called *sophismata* (the most comprehensive and widespread possibly being those of Kilvington and of Heytesbury).[2]

By the middle of the 14^{th} century, a new point of view emerged in the continent, especially in Italy and France, according to which many of the problems addressed by the calculators, and in general, many of the problems in natural philosophy, can be better approached using geometry as a method of representation. This method was found to be particularly fruitful for all problems involving intensive magnitudes. The concept of "latitude" became central in this context. It primarily refers to the range of intensive alteration of a quality, for instance for the heat of even a certain color. The intensity of these qualities can vary between a minimal and a maximal value – which was usually arbitrarily set for the sake of the discussion within a scale between, for instance, 1 and 8. To refer to this range of change or intensive variation, the notion of "latitudo" was normally used.

The Main Sources of the Geometrical Approach

The main texts representing this new approach are, first, the *Questio de velocitate* by Giovanne da Casale (and of it, only a brief passage), then, also the questions on Euclid's geometry (QGE) by Oresme and his treatise *De configurationibus* (DC), and finally two treatises which in the current scholarship, following the research done by Anneliese Maier, are attributed to one and the same author, the *Tractatus de perfectione specierum* (PS) by Jacobus de Napoli and the *Tractatus de latitudinibus formarum* (LF) by Jacobus de Sancto Martino.[3] However, one needs to be warned about the fact that, although it is possible to identify Jacob de Napoli with Jacobus de Sancto Martino, this identification is neither sure nor obvious in itself.

Besides these questions referring to the authorship of LF and PS, many questions related to the dating and possible mutual influence between all these treatises has not been definitively settled yet. Thus, it is generally accepted that G. da Casale's question represents a first step in the development of these ideas, whereas PS and above all LF are a later, simplified version of Oresme DC. This reconstruction, accepted by both A. Maier and M. Clagett, is, however, by no means certain. As it has been objected, it is not impossible that the LF and PS were previous works that Oresme not only knew but also decided to correct and expand. Hence, it is understandable that Oresme DC includes far more relevant material related to this topic, which he could have added to improve this doctrine. That, especially, the author of LF would have simplified the doctrine to such an extent as to exclude the most important parts connected to the motion seems to be hardly acceptable.

The Perfection of the Species

However these questions are to be satisfactorily answered, the fact is that PS is a treatise that referred to a topic on which also other authors have written: the perfection of the species. This is a topic often addressed in some commentaries to the *Sentences* of Peter the Lombard, as for instance in the commentaries by Peter Ceffons and Alexander of Peter of Candia (the later Pope Alexander V). The core of the question is theological and involves the notion of "order," a notion which already had a long Augustinian tradition, but it also plays an important role within Aristotle's philosophy, especially as far as this speculative discipline was characterized as "theology" in the XII book of his *Metaphysics*. This link between theology and metaphysics opened the way, from the second half of the 14th century onward, to a full treatment of the problem of the perfections of the species within some commentaries to the *Metaphysics*, as for instance in the commentaries by Nicholas of Amsterdam and later by Paul of Venice.[4]

The problem addressed by Jacob de Napoli in this PS and in other similar texts is, in general, an old problem in Western theology and philosophy. Its significance for the history of ideas has been pointed out in the classical work by Arthur Lovejoy *The Great Chain of Being*, although with very limited consideration of medieval thought and complete exclusion of the quantitative approach represented by the calculators. Furthermore, Lovejoy was completely oblivious of the fact that there was a discussion about the way of representing the "great chain of beings." However, the way Jacob of Naples himself announced, at the very beginning of PS, how he would approach the topic, proves that by then it must already have been a well-disseminated topic. The leading question obtains its full significance only under some religiose suppositions of Platonic and Neoplatonic background: God, all-powerful and all-knowing Creator, has produced the most beautiful and ordered world. All beings belonging to it must have a "place" in this order of perfection which justifies their existence. Following the new approach of the 14th century, according to which quantitative aspects gain in importance, perfection (*perfectio*) can be understood as an intensive magnitude, which allows at least a mathematical treatment of the problem. The new idea is now to introduce the concepts of degree and of latitude: "Perfections" can be quantified by attributing degrees to them. By so doing we can establish the range of perfection, the *latitudo perfectionis*, which belongs to a certain entity and we can even compare one to the others.

The introduction of a mathematical approach is, nevertheless, not unproblematic. For, at the same time, the problem of continuity is involved: if God has created nothing without a reason, he must also have created all that was necessary.

The action of a perfect God does not allow gaps. On the basis of these general premises, which are difficult to reject for a 14th century philosopher, Jacob discusses above all two opinions: one arithmetical and the other geometrical. According to the first, the order of entities in the world, the scale of being, can be represented using numbers, starting with 1 for the lowest degree of perfection and increasing to infinity, the maximum perfection of God. This thesis, however, violates the postulate of continuity because, according to the current conception of numbers, between two (natural) numbers there is no other number (at best a relation, or proportion between them). Jacob decides himself for the geometrical thesis, the main idea of which consists in representing the latitudes of perfections by perpendicular lines to other lines used as basis of the diagram. As the perfections grow from the simpler or "less perfect" beings to the highest levels, the lines grow longer. The advantage of this conception over the arithmetical one is that the postulate of continuity can be respected, since the latitudes are made up of degrees, they are continuous magnitudes and so, between two arbitrarily taken degrees of perfection, a third degree can be introduced. This conception is clearly more intuitive than the arithmetical one and it makes clear that, by using geometrical entities, no gaps can arise.

As a matter of fact, Jacob dedicates the second chapter of PS to briefly present some of the main notions concerning the topic of the perfection of species (perfection of the species, *perfectio* according to the essence, accidental perfection, gradual *perfectio*, etc.), and the "basic geometry" needed for representation (point, line, straight line, superficies, different angles).[5] PS was never printed, however, it is conveyed in eleven manuscripts. Moreover, a later Nord-German copy from the 15th century includes a commentary on it. As for many texts of this kind, it is not always easy to decide which figures belong to the text itself, as composed by its author, and which are additional work made by the scribe clarifying the content of the text. In this copy of text preserved in a Munich manuscript (BSB, clm 26838 – **Image 24**), the scribe has clearly depicted in the margins some of the pure geometrical notions (the different angles and, at the bottom right, two squares used to demonstrate the irrationality of the diagonal of the square to its side) and two representations of the perfection of species: One of them (on the right side, in the middle) is an inverted isosceles triangle in which the different degrees of perfection are noted at both sides growing from non-degree (*nullus gradus*) according to the series of even numbers: 2, 4, 6, 8, 10. The parallel lines crossing the triangle also become longer as their distance from the triangle's vertex increases. The other representation (on the bottom left) uses more accurately a right-angled triangle on the basis from which the perpendicular lines are erected. The reader of the text is now able to visualize how the perfection of beings is growing from "no-degree" until the *summus gradus* (occasionally, it is worth noting that the treatment of the perfections of the species often included a special discussion regarding the issue of whether the *summum* or *maximum* belongs to the series – like in the example represented here –, or remains out of it since God cannot be placed on the same series with His creatures). The many "gs" on the basis refer to "degree" (*gradus*). The hypotenuse features the numbers of the highest degree for each of the represented latitude. Between *non gradus* and *summus gradus*, the numbers grow according to an equal excess of two, hence it has to be a straight line which is not, however, parallel to the basis (under this diagram a further graphics was added by the scribe which seems to have been speculating about the differences of the degrees and how to represent them). Like other authors, Jacob distinguishes first between a "uniform" and a "difform latitude." In the first case, all intensities have the same degree and so all perpendicular lines representing them are equal. In the second case, the intensities vary and so also the representing lines. Here are two cases still to be distinguished: when the variation is in itself uniform, one speaks of a "*uniformly* difform" latitude; when the variation does not remain regular but it varies itself, we have a "*difformly* difform" latitude. These two figures included in some copies of PS visualize the notions accordingly. It is important to note that Jacob uses the term *latitudo* to mean not only the single lines erected parallelly but also the whole, complete figure itself. The thesis he supports is that the best method of representing the "series of beings in the Universe" consists in using a triangle for *uniformiter difformis* – **Image 26**.

The Latitude of Forms

It is out of the question that the treatise PS contains a geometrical approach in the treatment of the intensive magnitudes. Jacob of Naples, however, restrains himself to a basic presentation of the techniques of visualization.

These are much more explicitly analyzed in the treatise LF (Ed. Smith, 1954), whose author has not been clearly established but which can also be attributed to him (in several manuscripts and printed editions the text is wrongly attributed to Nicole Oresme). This treatise had an enormous influence in the second half of the 14th century and, especially, in the 15th century. It was copied more than fifty times (!), commented on, summarized, and integrated into other texts.[6] Especially in German universities, first of all, at the Vienna's Faculty of Arts, it belonged to the teaching material students used to learn along with other "middle sciences", like astronomy, optics and the proportional analyses of motion, and for the sake of a better understanding of *sophismata*.[7] Also in Italy, this text experienced a wide interest. Among others, it was thoroughly studied by the famous philosopher and logician Biagio Pelacani da Parma, who wrote a set of three questions on it.[8]

LF is also a short treatise, made up mainly of three parts. In the first one, the author gives his division of the *latitudines*. Here, to the aforementioned distinction between uniform and difform, the latter being divided into uniform difform and difformly difform, he adds a further division for difform latitudes. The rate of change of a difformly latitude can itself change according to a certain regularity, and then we would have uniformly difformly difform latitude, or change without such regularity in whatsoever way, and, in this case, we would have a difformly difform difform latitude. In addition, LF is quite explicit when trying to contribute to a visualization of the problems connected to the first and the last instant of change (*de primo et ultimo instanti*). To this end, the text includes a further distinction referred to both extremes of the figure (and correspondingly) of the latitude which could respectively be representing a certain degree (*ad gradum*) or no degree (*ad non gradum*). Of course, of the four possible alternatives, only three would produce a sensible representation (at least one extreme must be *ad gradum*; both extremes with no degree would produce a representation with no perpendicular lines). Besides, of course, a latitude could be uniform or difform in whatever form in only a part. As one can appreciate in **Image 25**, the old printed edition from Padua (1482, by Matheus Cerdonis) includes, as several manuscripts as well, representations of the last-mentioned cases on the right margin: Reading the figures from left to right, the figure on the top is a latitude "starting with no degree" (*incipiens a non gradu*). The figure under it shows a latitude beginning with a certain degree (*incipiens a certo*) and – what it is not included in the verbal description – also ending with a certain degree. Both figures are uniformly difform. The next figure is a section of a circle representing a beginning with no degree. If we now cut the one extreme with a vertical line, we obtain a similar latitude which, however, has a degree at the beginning and at the end (*incipiens et terminata ad gradum*). The author of LF observes that his system can be applied not only to permanent things (*permanentia*), as it is understood for the distribution of a quality on a body, but also for successive things (*successiva*), like the development of an intensive change in time, i.e. to motion. This distinction is at the core of the more deep and original works in this tradition, composed by Nicole Oresme.

Oresme's configurations doctrine

From Nicole Oresme's work on natural philosophy, it is evident that he knew many of the so-called analytical languages. In his early questions on Euclid (esp. qq. 10–16) he explored this method or representation and proposed a first version of this new doctrine which will reach its more developed expression in the Treatise *De configurationibus qualitatum et motuum* (DC).[9] If it is true that Jacobus of Sancto Martino is the same person as Jacob of Naples, and we are allowed to identify this author – from whom no other works are known – with Jacobus Caracciolo de Napoli, who died in 1357,[10] then, it seems highly probable that Oresme's configurations doctrine is a further development of the ideas previously expressed in PS and LF.

Oresme's doctrine was never printed. According to the current state of knowledge, the first text is conserved in only the fourth manuscript, whereas the second one is extant in fifteen copies (Clagett's edition is based on the fourteenth manuscript). A further, important manuscript has been recently reported[11]: Metz, Bibliothèques-Médiathèques, MS 378, ff. 1r–58r). Oresme pointed out in the foreword of this work that he intended to offer the principles of a new discipline and not only a textbook with exercises. Besides, he emphasized that what he in general seeks is to obtain clarity in those matters which others have treated confusedly. As the third part of this treatise, where he comes to apply the pure geometrical doctrine to cases of motion, is undoubtedly connected with some previous *sophismata* discussed by the Oxford calculators, we can assume as highly probable that a good part of Oresme's project consisted in replacing the verbal discussions carried out in treatises on logic and collections of *sophismata* by geometry. It must be noted that, unlike logic, geometry was already an accepted and independent discipline. Besides, geometry was paradigmatic not only in the art of delivering proofs applicable on different fields but also, and most especially for the Oresmian project, for its visualizing possibilities.

Oresme points out, however, that the main idea of this project is not only to represent but to visualize intensities (*intensiones*) that are not perceptible by means of expressing them in terms of extensions. He placed much emphasis on a clear distinction between the geometric terms used and the ontological realities to which they refer (something less clear in LF). Limiting the explanations to the case of representing only in two dimensions, one could first set the geometrical entities, points, lines and surface to be used.[12] These entities, however, are not independently existing entities. There is nothing like "a point," or "a line" or "a surface" in itself, like in the Platonic philosophy of mathematics. We then go on to construct the figures (*configurationes*), paying particular attention not to confuse the geometrical terms themselves with the physical or ontological terms to which they refer (let us use the arrow $\longrightarrow$ to clarify this connection). A basic principle of the representation requires that we always proceed proportionally, i.e.: one can take any line to represent an intensity, but then, to represent an intensity twice as long, one must represent a line twice as long. Obviously, the requirement of proportionality must also be respected for the "longitude" representing extensions (in body or in time). An ontological assumption – which occurs in both LF and DC – states that the representation cannot accept angles greater than a right angle (otherwise we would be proposing a quality without a subject). The highest points of the latitudes representing the intensities represent, in turn, the maximum degrees of each intensity. These degrees can be united in a line called by Oresme *linea summitatis*. The total quantitative result of an intensive quality that has been represented in such a way can be seen in the "quantity of the quality" represented on the surface of the figure – **Image 27**.

Using this method of representation, Oresme achieves great clarity in the exposition of cases verbally explained by the Oxford calculators, such as the famous “mean speed theorem” and several particular cases involving the increase of the velocity in *infinitum*.

Finally, it is worth remembering that, as Maier has pointed out, Oresme’s intention was not to reduce his doctrine to a mere method of representation (like the theory expounded in LF). He was convinced that his theory was capable of grasping, at least *secundum imaginationem*, an aspect of reality not immediately open to the senses, the world of intensities.[13]

Notes

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1 For an overview see Sylla (1982).
2 For the notion of “languages of analysis” or “analitical languages” see Murdoch (1975 and 1981).
3 See A. Maier (1952), pp. 371-75. Jacobus of Naples is sometimes called “James” in the bibliography. In this pages, I shall always refer to him as “Jacob”.
4 For the introduction of calculatoy notions and diagrams in commentaries on Aristoteles’ *Metaphysics*, see Di Liscia (2022).
5 See Maier, loc. cit, Di Liscia, loc. cit.
6 For a list of manuscripts and old printed books containing LF Di Liscia (2003), pp. 421-29.
7 Di Liscia (2016).
8 Di Liscia (2019).
9 An excellent critical edition of this treatise was prepared by Clagett (1968). A more recent edition of Oresme’s questions on Euclid was published by Busard (2010).
10 Zumkeller (1966), p. 204 (mentioning only five copies in four manuscripts).
11 Di Liscia (2017).
12 Oresme clearly notices that also, under certain circumstances, it is possible to extend the method of representations to three dimensions (see DC, III.1, Ed. Clagett, p. 270-271).
13 See especially Maier (1968), pp. 89-109.

7. Celestial Bodies and Dimension of the Universe in Bartolomeu Velho's *Cosmographia*

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In the 15th and 16th centuries, the art of sailing, astronomical observation and maritime cartography reached a high level of development in Portugal, combining practical knowledge with the scientific knowledge of orientation by the celestial bodies and astronomical calculations.[1] Many cartographers drew the outlines of acknowledged space or newly discovered continents with detail and precision, thus reconfiguring the representations of the world. Maps facilitated travel, broadened knowledge, and, above all, documented dominion and sovereignty over distant territories. This art of representing the world has a precious and still innovative exponent in *Principio da verdadeira cosmographia et geographia uniuersal de todas as terras que são descubertas* (Principle of the true cosmography and universal geography of all discovered lands), known in a single manuscript, "feito per Bartholomeu velho portugues neste anno de 1568" (*made by Bartolomeu Velho, Portuguese, in this year of 1568*).[2]

The large format manuscript book (c. 335 x 235 mm, paper) contains astronomical tables, planispheres, tables with calculations of the movements of the moon and other planets, calculations of the dimensions and distances of the celestial orbs from the center of the earth to the external border, which show the most advanced geographic and cosmological knowledge of the time. On folios 9v–10r, a polychrome representation integrates the Earth into the Cosmos using conventional symbols and graphic forms, to which calculations of the dimensions, distances and relative motion of the celestial bodies are innovatively added – **Image 29**. The inclusion of quantitative data sets this scheme apart from the history of representations of the Aristotelian-Ptolemaic Cosmos[3]. Although later than Nicolaus Copernicus' *On the Revolutions of the Celestial Spheres*, published in 1543, the heliocentric hypothesis is not contemplated by Bartolomeu Velho, but astronomical observations play a rather innovative role in his work.

Bartolomeu Velho was born in Lisbon in an unknown year and died in Nantes on February 20th, 1568. The Portuguese crown's hydrographer and mathematician was too an important cartographer who, at the end of his life, got involved in an adventure of espionage and scientific knowledge trafficking, of which this manuscript is the most direct testimony. A quire in a different paper, bounded after folio 31v, with folios numbered 1 to 4, contains a letter and report written in French[4], simply addressed *Au Roy* (certainly Charles IX of France). In this letter, Bartolomeu himself tells of leaving his homeland, family and possessions to serve the king and introduce him to "les parties des terres incognues qui sont de grand importance et consequence" (f. 1*r). These unknown lands are never named or indicated and all the lands that are represented in the globes of ff. 9v–10r, or in the map projections of f. 10v, or in the globes that in different perspectives decorate the tables on ff. 6r–8r, were well known to cartographers at the time. The only large continental masses that are not represented are Australia and Antarctica. It would not be until 1606 that news of Australia would be brought by Dutch sailors, but here there is nothing to indicate that Bartolomeu knew of or anticipated its existence. It is more likely that these "terres incognues" (unknown lands) are, for example, in the continental interior of the New World, at the time not yet occupied or colonized by European powers. The letter goes on to state that new navigations will be needed to reach these lands, so Bartolomeu informs the King of the instruments needed for these voyages, describing others that he will develop himself if the King approves them (f. 1*r). The memorandum goes on to list 49 articles that include universals (planispheres), globes, charts and navigational processes or other rare instruments, necessary and useful "pour la vraye nauigation" (1*v–4*r). *Cosmographia* is a scientific testimony, prepared under the banner of truth (note as well the title of the work: *Princípio da verdadeira Cosmographia*), organizing scientific, technical, and practical knowledge that will allow the sovereign to make discoveries and take possession of unknown lands.

The secret escape and transfer of the cartographer and cosmographer from the service of King Henry the Cardinal and the Portuguese Armada to the service of the King of France in 1567, with the intervention of Italian agents from Luca, confirms the economic and military importance of Bartolomeu Velho's cutting-edge scientific knowledge[5]. Other works by Bartolomeu Velho are known: the four 1561 charts that cover the whole world and are therefore improperly considered a planisphere or *Carta General do Orbe*[6]; an anonymous chart in spindles that uses similar criteria to other works by Bartolomeu[7]; the anonymous 1560 Atlas that contains a boundary of Japan and uses distinctive projections, which led to it being attributed to Bartolomeu Velho[8]; and he is probably also the author of the "1148" map of c. 1560[9]. Well informed of Bartolomeu Velho's knowledge, the brothers Francesco and Andrea d'Albagno, Italians from Luca and agents interested in French sea voyages, manage to convince Bartolomeu to serve the King of France and inform him of unknown lands, passing on all the instruments and knowledge needed for the art of sailing to reach those lands.[10] By the end of September 1567 Bartolomeu is already in France, which means he would have taken the codex dated 1568 with him from Portugal. He died soon after in Nantes, on February 20th of the same year.

Cosmographia is incredibly thoughtful and was certainly intended to show the value of the knowledge that Bartolomeu Velho placed at the service of the King of France. It is clearly not a practical manual for sailors, although the contents would be of great use when navigating within sight of the coast or on the high seas with the help of astronomical observation. Even its descriptive and scientific elements are written in Portuguese, which would have made it difficult for French readers to use. However, the letter to the King and the instrument report in the final pages was translated into French, emphasizing and praising the quality, value and importance of the instruments Bartolomeu proposed to deliver or build.

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The *Figura dos corpos çelestes* (Scheme of the Celestial Bodies) that occupies folios 10v–11r – **Image 29** – is remarkable in integrating ancient cosmology, geographical knowledge, the new practical science of navigation and detailed astronomical calculations. Two tables with complex astronomical calculations contain the *Regimento da declinação do sol* (Regulation of Sun's declination) for a cycle of 4 years, in bifolio 3v–4r, and the *Revolução da Lua em vinte e sete dias e oyto oras, de 1568 a 1600* (Moon Revolution in twenty-seven days and eight hours, from 1568 to 1600), in folios 4v–5r – **Image 28**.

The *Figura dos corpos çelestes* is, first of all, a representation of the chemical, or elemental, composition of the universe according to the Aristotelian model. The sublunar world consists of four elements: earth and water represented on the surface of the two terrestrial hemispheres, surrounded by the sphere of air, represented in the form of winds and clouds, fire in its sphere is represented by small ascending threads and yellow background. Above that sphere there is no representation of the single celestial element, homogeneity and unity that, according to that elemental model, prevents generation and corruption, which only exist in the sublunar world, composed of 4 elements.[11]

This cosmographic map presents a perfectly circular universe,[12] with the fold line of the bifolio allowing the opening of a wide column that separates the two hemispheres and where the caption is inserted. This separation of the two hemispheres also enables the representation of the Earth's globe in two faces, each one with continental masses represented with different colors, with much cartographic detail despite the reduced size. The equator lines, the tropics and the equinox, especially the curvature of the meridians, give the planar representation the appearance of the Earth's sphericity. On the right half globe are Europe, Africa and Asia, on the left half globe the northern and southern New Worlds, flanked by the Ocean Sea (Atlantic) and the Pacific Ocean, with scattered islands and the coast of Asia and the Japan island already standing out.[13]

The Earth is surrounded by a circular ring of clouds (the Air), itself surrounded by another ring with ascending filaments (the Fire). After the spheres of the four elements, each ring or circle corresponds to a celestial body. On this chart, the celestial bodies are not represented, except in the case of Firmament, with a myriad of stars depicted in gold. Instead, at the end of each semicircle, i.e., 4 times, there are the conventional signs representing the Moon (full Moon and waning Moon), the Sun and the stars, with Mercury, Venus, Mars, Jupiter and Saturn represented by their zodiac symbol.

Instead of representing the celestial bodies, each circle includes the name of the sphere and a multitude of quantitative data arranged in columns like radii of circles. The name of the sphere appears at the bottom right-hand side in block capitals, also used at the top of each sphere to indicate its revolution (i.e., the apparent motion "around" the Earth), 27 days and 8 hours for the Moon, to 36,000 years for the stars' sky, or the expected 24 hours for the sphere of the First engine, the air and fire spheres being devoid of rotational motion, as they are sublunar and therefore immobile like the Earth.[14]

The cosmos includes the following mobile spheres: Moon, Mercury, Venus, Sun, Mars, Jupiter, Saturn, Stars or Firmament, *Primo mobilem*. And above these a motionless sphere: the *Habitaculum Dei*.

It is a full universe, filled, compact, of perfect concentric symmetries where Bartolomeu relinquished the representation of the very cause of each body's movement and its celestial sphere.

The cause of the celestial bodies' movement and elements is identified in their remote cause, the *Primo mobilem* or first motor, which has a 24-hour revolution. On its exterior is the transcendent sphere, represented in red and which is the *Celum empireum habitaculum Dei et omnium electorum* (Heavenly empire, dwelling of God and of all the elected). For this sphere no revolution time is indicated and by its primacy it has the place of the unmoved mover. In Aristotle, the first unmoved mover is a natural and eternal cause of motion (see Aristotle, Metaphysics, XII.7–8; 1072a18–1074b14), in Bartolomeu Velho, the single unmoved mover is the Empyrean Sky, thus integrating, as was also traditional, this theological sphere into the Aristotelian natural world. Its supernatural nature is indicated by immobility and by the figuration of a compact circular choir of winged angel heads, all turned and looking into the celestial orb. The first cause of movement and light, the cause of the world, is symbolized in the four corners of the map: in the lower corner two light sources radiating into the universe, in the upper left corner the Crucified One and the Holy Spirit represented as a dove, in the upper right corner God the Creator, with white hair and beard, with arms and hands open in a gesture of gift and protection of the Orb.

To the Aristotelian-Ptolemaic natural world, reduced to its chemical composition and geometric structure in concentric circles, the creationist and supernatural element of the Christian religion is added, placing above the first moved mover the Empyrean, or the dwelling place of God and God's elect, the only sphere not by chance to be named only in Latin.

The quantitative data gathered and inserted by Bartolomeu in the *Figura* are impressive and are further detailed and explained in other representations and tables in the manuscript. The dimensions and distances are in *legoas* or *léguas* (or Portuguese miles, c. 6.6 Km each *légua*), with 6,300 Portuguese *legoas*/miles for the circumference of the Earth. Clockwise we see the extensions for each sphere, exemplified here in each series only with the Star sphere and the Moon sphere: (1) *Léguas de um grau em cada globo celeste* (Portuguese miles of a degree in each celestial globe): stars sphere: 562,375 *léguas*; moon sphere: 910 1/3 *léguas*); (2) *Léguas do círculo maior de cada globo* (Portuguese miles of the greater circle of each globe): of the stars: 202,455,000; of the Moon: 327,707 2/7; (3) *Léguas do diâmetro de cada globo* (Portuguese miles of the diameter of each globe): of the stars: 64,417,500; of the Moon: 104,270 1/2; (4) *Léguas de grossura de cada globo* (Portuguese miles of the thickness of each globe): of the stars: 16,239,375; of the Moon: 24,876; finally *Leguas até ao centro da Terra desde cada globo celeste* (Portuguese miles from each celestial globe to the center of the Earth): of the stars: 32,208,750; of the Moon 52,135 1/4.[15] These numbers show the different dimensions and diameters of each celestial sphere (it would be better to say of each celestial layer), thus dispelling the idea of regular intervals between each celestial body, which traditional representations presupposed.

The universe world is made of relative proportions implying a graphic representation that would distort the *Figura*. That is why Bartolomeu Velho opts for this simple and uniform model, and the quantitative data indicate to the expert that the image we see does not correspond to the dimensions and proportions of what would be the real cosmos, in which the spheres should rotate at different speeds.

The images in the following folios unfold and explain what is here in condensed form. For example, folio 10v contains a general chart of the Earth's surface in two semicircular projections for the Northern and Southern hemispheres wrapped in air, thus completing the double representation of the globe, and in f. 11r a schematic representation of the proportion of the parallels on Earth's globe. Next, an open bifolio is dedicated to each planet, on the left with the mythological representation of the planet's God or Goddess, its chariot and the animals that pull it and make each sphere move, and on the right the proportional representation of the parallels in the sky globe of that same planet, from the Moon to Saturn (ff. 11v–18r). Each sphere's movement-cause is represented by the god who gives the name to the planet. The Empyrean of the Stars has the proportional representation of the parallels, but instead of a god moving it, is represented how the stars are observed with a quadrant (f. 18v–19r).

One of the most notable innovations of this *Cosmografia* is found in folios 19v–22r with the representation of longitudinal sections of the universe, from the center of the Earth to the outer limit of the *Primo Mobile*. Instead of reducing the dimensions to the proportion of the bifolio, Bartolomeu Velho extends the sheets to show the Cosmos dimension in more detail:

– Folio 19v is of a normal size and extends into the much longer folio 20r, with the open bifolio measuring 335 x 2995 mm, to offer an elongated representation of the Earth with the Old World, wrapped in the successive spheres of the elements Air and Fire, with the Moon Sky to the Mercury Sky.

– The bifolio 20v–21r has an approximate size (335 x 2760 mm) and the same representation, from the Mercury Sky to the globe of the Earth, showing the New World. In both cases the representation is traversed in the center by a numerical scale that gives the distances in *léguas*/Portuguese miles.

– Bifolio 21v–22r – **Image 30** – is also foldable (21v with the normal size, 22r wider, at 335 x 2995 mm in total), with the "*demonstração em proporção por legoas*" (demonstration in proportion by Portuguese miles) further reduced, as evidenced by the scale in the center and by going to the first Sky of the *primo mobile*. It includes the depiction of the planet's spheres, with epicycles and eccentrics, and beautifully detailed small polychrome and gold images of the gods, chariots, and animals that move each planet.[16]

The three extendable representations have a fine gold frame, letters and polychrome representations, with the use of silver and gold. These schemes provide a new view of the structure and size of the Cosmos.

The *Figura dos corpos çelestes* (ff. 10v–11r) and the *Demonstração em proporção* (ff. 19v–22r) propose spectacular visualizations of the geocentric and elemental Cosmos but are functionally different and show very different stages of scientific knowledge of nature. *Figura dos corpos çelestes*, despite the novelty introduced by Bartolomeu with the multiple quantitative data and the representation of the continents on the central globe, still visually offers the closed, compact, disproportionately homogeneous Cosmos of the Ancient and Medieval tradition. The three successive *Mostrações do Cosmo nas suas proporções* (Representations of the Cosmos on their proportions) visually represent the Cosmos from the new data of the science of astronomical observation, undoing proportions, regular movements and symmetries, thus contributing to the imagination of a larger world, impossible to embrace by sight, requiring successive approximations and representations to access the details. It is science helping to expand the horizons of the world, while inventing new visual methods to aggregate large amounts of data, also numerical.

The grandiosity, geometric rigor, and chromatic beauty of Bartolomeu Velho's universe has a poetic parallel in the tenth and last *canto* of the *Lusiadas*, the great epic poem by Luiz Vaz de Camões, published in Lisbon a few years later, in 1572.[17]

As he prepares to return from the arduous voyage of the discovery of the sea route to India, Vasco da Gama and his companions are rewarded by the goddess Thetis, who takes them to a high point to contemplate the grandiose Machine of the World.[18] From there they see a transparent globe suspended in the air, composed of several concentric orbs. It is a perfect globe, such as the *Archetype* which created it (X.77–79). In this *minor mundus*, Gama and the Portuguese argonauts can see the journey they will make on their return (X.79.6–9). They see a detailed artificial model of the World, created by God itself:

> Vês aqui a grande máquina do Mundo,
> Etérea e elemental, que fabricada
> Assi foi do Saber, alto e profundo,
> Que é sem princípio e meta limitada.[19]
>
> Here see the mighty World-machine appear,
> ethereal where the fourfold elements blend,
> made by His deep design, His lofty lere,
> who lacks beginning and who has no end.[20]

The goddess Thetis remains silent and, seeing the model that aggregates the creator and the created world, the poet, through the Gama, proceeds with describing the orbs, from the outside to the inside. First the theological spheres, the *Empyreum* or God who creates and surrounds everything (X.81–84) and the *Mobile primeiro* (85–86), treated at different lengths for the details of their designations and nature. Below this is the wide Firmament, the sphere of stars and constellations (X.87–88) and in a single octave condenses all the other orbs: Saturn, Jupiter, Mars, Sun, Venus, Mercury, Moon (89.2–8), finally the elemental world occupying the center of the World Machine (90.6–8). Following then a detailed description in 408 verses of the earth's surface, "pousada dos homens" (*inn of humankind*), as if a vast and detailed map were being described (X.91–141).

The poet thoroughly describes cosmographic and geographical representations that cartographers like Bartolomeu Velho presented with increasing detail and rigor in globes or planispheres.

To these condensed representations of knowledge, we can apply the paradoxical words with which Thetis justifies to Gama wanting to take him to contemplate the Machine of the World:

> Faz-te mercê, barão, a Sapiência
> Suprema de, *cos* olhos corporais
> veres o que não pode a vã ciência
> dos errados e míseros mortais.[21]

To thee Supremest Wisdom guerdon gave,
Baron ! who hast beheld with fleshly eyne
what things the Future hath the power to save
from Mortals' petty pride and science vain.[22]

Camões' Machine of the World, a gift of the Supreme Wisdom, enables a vision by sensory experience of what mortal science cannot reach. As the poet truly knows, the representation of the Machine of the World is nothing but the science of mortals that makes it possible to embrace by sight and in a single figure the vast universe world.

Notes

1 Cf. *Portugaliae Monumenta Cartographica*, 7 vol., (Eds.) Cortesão, A. and Mota, A. T., Comissão das Comemorações do V Centenário da Morte do Infante D. Henrique, Lisboa 1960–1962. For an updated overview, see Gaspar, J. A. (2021). Navegação e cartografia náutica nos séculos XV e XVI. In Simões, Ana – Diogo, Maria Paula (coord.), *Ciência, tecnologia e medicina na construção de Portugal: Novos horizontes*. Volume 1: *Sécs. XV – XVII*, Tinta da China, Lisboa, pp. 101-126; Sánchez, A. (2021). Cosmografia, náutica e conhecimento prático: espaços, prática e artefactos. *ibidem*, pp. 127-150.

2 Ms. Paris, Bibliothèque nationale de France, département Cartes et plans, Rés. Ge. EE 266, available in Gallica: https://gallica.bnf.fr/view3if/ga/ark:/12148/btv1b550128121/. See *Portugaliae Monumenta Cartographica*, vol. II, pp. 103–105, plates 205–210. For the first time described in Viterbo, F. S. (1898). *Trabalhos náuticos dos portugueses: séculos XVI e XVII*, Parte I *Marinharia*, Academia das Ciências, Lisboa, vol. I, pp. 310–321, when it was owned by the Portuguese bibliophile Jerónimo Ferreira das Neves, the manuscript then disappeared for some decades and was bought in 1950 by the Bibliothèque nationale de France at the auction of Charles Chadenat library.

3 Cosgrov, Denis E. (2007). Images of Renaissance Cosmography, 1450–1650, in *The History of Cartography*, vol. 3.1: *Cartography in the European Renaissance*, ed. David Woodward, The University of Chicago Press, New York – London, plate 2 ("Perhaps the most accomplished sixteenth-century attempt to map the scale of the world machine") and pp. 83, 84 on Bartolomeu. On Early Modern developments of this science, see Vogel, Klaus, A. (2006). Cosmography. In Parker, K. – Daston, L. (eds.), *The Cambridge History of Science*, vol. 3: Early Modern Science, Cambridge University Press, Cambridge, pp. 469–496.

4 Edited in Viterbo, *Trabalhos náuticos dos portugueses*, cit., pp. 315–321.

5 Biographical elements in *Portugaliae Monumenta Cartographica*, vol. II, pp. 89–92 and Mota, A. T. (1966) O cosmógrafo Bartolomeu Velho em Espanha, *Revista da Faculdade de Ciências*, 39, offprint.

6 Ms. Firenze, Accademia di Belle Arti; cf. *Portugaliae Monumenta Cartographica*, vol.II, pp. 95–101, plates 201–204.

7 Ms. Paris, Bibliothèque nationale de France, Rés. Ge D 7824; cf. *Portugaliae Monumenta Cartographica*, vol. II, pp. 93–94, plate 200.

8 Ms. San Marino, Calif., Huntington Library and Art Gallery, HM 44; cf. *Portugaliae Monumenta Cartographica*, vol. II, pp. 107–109, plates 227–236.

9 Ms. Paris, Bibliothèque nationale de France, Rés. Ge B 1148; cf. *Portugaliae Monumenta Cartographica*, vol. I, p. 165, plate 84.

10 The kings Henry of Portugal, Philip II of Castile, and Charles IX of France are involved in the arrest of Andrea d'Albagno in Seville, one of the accomplices in Bartolomeu Velho's escape, which shows how important Bartolomeu was to all of them, see Mota, O cosmógrafo Bartolomeu Velho em Espanha, cit.

11 The theory of the elements has a central place in Aristotle's philosophy of nature, which he develops in several of his works, all widely commented on and assimilated in medieval scientific culture: *Meteorology* (I.2 on the two parts of the orb, sub and supralunar), *On the Heavens, Physics, Metaphysics*.

12 On the origins and medieval popularity of the spherical universe, see Obrist, B. (2020). The Idea of a Spherical universe and its Visualization in the Aerly Middle Ages (Seventh to Twelfth Century). In Kupfer, M., Cohen, A. S. and Chajes, J.H. (Eds.), *The Visualization of Knowledge in Medieval and Early Modern Europe*, Turnhout, Brepols, pp. 229–258; and for the major Medieval sources, major problems and innovations, see Grant, E. (2013). Cosmology. In (Eds.), *The Cambridge History of Science*, vol. 2: Medieval Science, Cambridge University Press, Cambridge, pp. 436–455; North, J. Astronomy and Astrology, *ibidem*, pp. 456–484.

13 This and other Earth representations are already distant from the imagined cartography and symbolic cosmology of the medieval period, about which it is also worth seeing Edson, E. and Savage-Smith, E. (2004). *Medieval Views of the Cosmos. Picturing the Universe in the Christian and Islamic Middle Ages*, with a foreword by Terry Jones, Bodleian Library, Oxford.

14 On the spheres, astronomical measurements and the influence of works such as Ptolemy's *Almagest* and the *Treatise on the Sphere* by Johannes de Sacrobosco, see again Grant, "Cosmology", cit., and North, "Astronomy and Astrology", cit.

15 The cosmogram condenses the tables of the previous folios, including: quadrennial tables of the sun's declination, with height rules and by the Southern Cross (ff. 3v–4r); tables of the moon for the years 1568 to 1600 and the table of tides to calculate the high tide on the "Costa braba" (4v–5r); table of movable feasts, *letra dominical* and *número áureo* for the years 1568 to 1600 (5v); table to know the size and thickness of each celestial globe, in miles and Portuguese miles (6v); table of Portuguese miles, diameter and greatest circumference of each globe (f. 7r); table of the degrees of circle of each celestial globe (7v); table of the degree of greatest circle on each celestial globe and how many Portuguese miles it has (8r); table of the diameter of the earth in Portuguese miles and the distance to any celestial body (8v); table of the hours and minutes of the greatest day of the year at any latitude in the northern hemisphere and southern hemisphere (9r).

16 According to Cosgrove, these representations derived from Erhard Ratdolt's astrological illustrations, cf. Cosgrove, "Images of Renaissance Cosmography", cit., p. 83, n. 119; they appear too in Andreas Cellarius' *Planisphaerium Ptolemaicum siue machina orbium mundi* (in *Harmonia Macrocosmica*, ed. Johannes Janssonius, Amsterdam 1661, plate 1), that in the details has more differences than similarities with Bartolomeu's celestial plan.

17 Camões, L. (1572). *Os Lusiadas.* (Ed. Antonio Gonçalvez, Lisboa), quoted by canto, stanza and verse: X.79.6, or by stanza and verse 79.6–8.

18 Silva, L. P. (1915). *Astronomia dos Lusiadas*, Imprensa da Universidade de Coimbra, Coimbra, on the Machine of the World, see Ch. VI, pp. 49–70.

19 Camões, *Os Lusiadas*, cit., X.80.1–4.

20 *Os Lusiadas (The Lusiads)*, 2 vol., Englished by Richard Burton, ed. by Isabel Burton, Bernard Quaritch, London 1880, X.80.1–4, p. 390.

21 Camões, *Os Lusiadas*, cit., X.76.2–4.

22 *Os Lusiadas (The Lusiads)*, Burton's version, cit., X.76.2–4, p. 393.

8. Diagrams and Drawings in Charles S. Peirce: Reasonableness and Creativity

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Introduction

The American philosopher and scientist Charles S. Peirce (1839–1914) considered diagrammatic reasoning as the possibility of correct and fruitful reasoning. That kind of reasoning meant placing a sensitive element at the center of our reason, an element that allows us to understand, connect, and improve our ideas. Making a diagram was for Peirce much more than giving concepts a visual form. The use of diagrams and graphic elements is not something auxiliary, a mere transcription of thought into drawings; instead, diagrams are seen as processes that organize thought in its space–time arrangement, which is just what allows us to understand and develop our thinking. The arrangement of the parts gives us "facile mastery of far more complicated forms of connection" (NEM 4, xx, 1911), to the point that diagrams become one of the keys to explain creative processes.

In this text, we will first approach Peirce's notion of diagrams; then, we will focus on the role of diagrammatic reasoning in the Peircean notion of creativity. Finally, we will take as an example the illustrations and drawings used by Peirce himself in letters of his European correspondence.

Peirce's Notion of Diagrams

Throughout his many works, Peirce repeated on numerous occasions that diagrams illustrate the general course of thought (see, for example, CP 4.530, 1906). This idea corresponds to his belief that reasoning is not a mechanical function, nor a closed mental faculty. Peirce's notion of reason, which is very distinct from the isolated conception of reason derived from rationalism, may be called "reasonableness."[1]

Peirce's notion of reason is not something which dissects problems, nor it is merely consciousness. The essence of reason lies for Peirce in thirdness, allowing us to connect things together, to compose (cf. CP 6.343, 1908). For its development, reasonableness depends upon feelings, imagination, and instinct, and it relies upon a broader way of thinking that may even be illustrated in our minds and in our imagination with drawings and diagrams, as he wrote in his "Neglected Argument for the Reality of God." Here, Peirce mentioned a particular occupation of the mind that he called Musement. This peculiar activity, which consisted in letting the mind wander without rules or purpose, was at the root of all reason, and was, for Peirce, of extraordinary fertility. This is the activity, so contrary to what is sometimes meant by "rational", in which logical analysis, according to Peirce, can achieve its full efficiency. Peirce wrote:

> Enter your skiff of Musement, push off into the lake of thought, and leave the breath of heaven to swell your sail. With your eyes open, awake to what is about or within you, and open conversation with yourself; for such is all meditation. It is, however, not a conversation in words alone, but is illustrated, like a lecture, with diagrams and with experiments (CP 6.461, 1908).

But, what is a diagram according to Peirce? A diagram is for him an arrangement of a set of rationally related objects. For Peirce, the diagram has an iconic nature, that is, it is a sign that "represents its object in resembling it" (CP 6.471, 1908). In the case of the diagram, the similarity shown is the relationship between the parts of something: the function of a diagram is to represent relations of the parts of the object or objects through visible relations analogous to them (CP 4.433, c.1903). Therefore, a diagram is an icon of intelligible relations that must be analogous to the relations in the object (CP 4.531, 1905).

According to Peirce, such similarity does not imply a physical or sensuous resemblance, but only an analogy between the relations of the parts of each (CP 2.279, c.1895). The icon, on the other hand, does not tell us whether its object is real. Peirce states: "A sign may be iconic, that is, may represent its object mainly by its similarity, no matter what its mode of being" (CP 2.276, 1903). Peirce explained in 1905 that a diagrammatic system must be as simple as possible, that is, with as small a number of arbitrary conventions as possible, as iconic as possible, and as analytical as possible (CP 4.561, 1905).

On the other hand, diagrams are for Peirce the central part of deduction, which is nothing but diagrammatic reasoning. Around 1896, Peirce explained deduction as follows:

> Deduction is that mode of reasoning which examines the state of things asserted in the premisses, forms a diagram of that state of things, perceives in the parts of that diagram relations not explicitly mentioned in the premisses, satisfies itself by mental experiments upon the diagram that these relations would always subsist, or at least would do so in a certain proportion of cases, and concludes their necessary, or probable, truth (CP 1.66, c.1896; see also CP 2.778, 1901).

Diagrammatic reasoning therefore consists in drawing a diagram of a hypothetical state of things and proceeding to observe it. This observation suggests to us that something may be true, and we formulate it with greater or lesser precision. We then proceed to investigate whether or not it is true. For this, a research plan is needed and, according to Peirce, it is the most difficult part because, for that, the most important characteristics of the diagram must be selected, and the appropriate abstractions must be introduced.

Thus, diagrammatic reasoning implies the development of a diagram – constructing a representation –, its observation and experimentation, and a correct analysis of the results (NEM 4, 47 and following; L 75, 1901). In this illustration with diagrams lies precisely the effectiveness of thought (CP 6.461, 1908), because it is difficult to go further without the visual component. It can be said that the visual representation contributes decisively to the development of thought, since it implies the comprehension of the structure and order of the parts, and that understanding allows – as previously stated – experimentation and reaching new consequences and generalizations. The law of the mind, says Peirce, in the department of science, of art, and of practical life, consists in this, that ideas connect themselves with iconic ideas, so as to make up sets (NEM 4, xx; MS 1008, 1911).

Diagrams and Creative Thinking

We will now take a closer look at the role of diagrams in creativity. For Peirce, the character of deductive – diagrammatic – reasoning consists in that it must be applied to all possible cases, but it is not incompatible with the imaginative study of the diagram that is formed: it rather requires it.

The soundness of diagrammatic reasoning and its disconnection from reality are not opposed to observation or creative imagination, on the contrary, it demands and needs them, perhaps even more than other kinds of reasoning, since the diagrammatic representation, that is, the iconic representation, of the facts is realized in the imagination: "We form in the imagination some sort of diagrammatic, that is, iconic, representation of the facts, as skeletonized as possible" (CP 2.778, 1901). Hence, diagrammatic reasoning requires "the perfect imaginability, on the one hand, and the extreme familiarity, on the other hand, of spatial relations" (CP 4.246, c.1902); it requires observation and experimentation on a diagram that we have created, so we know everything about its conditions (CP 3.560, 1898).

Perhaps it has not been sufficiently considered that deductive reasoning is also creative, that – although at first glance it might seem contradictory – there is something new in it. By making a diagram, we play with it, we approach it from new perspectives, and it is possible to detect unnoticed and hidden relations among the parts of the diagram other than those used in its construction (CP 3.363, 1885); we see its limitations and new possibilities arise. The hypotheses of the reasoner are creatures of his own imagination, but he discovers in them relations which sometimes surprise him (CP 5.567, 1901).

Peirce argues that, by visualizing ideas, by giving them a diagrammatic distribution, we better understand the organization of their parts and increase our capacity for analysis and synthesis; we better understand the structure and order of our conceptions. Diagrams are not just a passive representation or a transcription of thought into drawings, they are not a mere image or illustration; they are something active that generates new thought. Diagrams are processes that will allow us to examine relations, detect anomalies and, ultimately, understand and make our reasonings grow. Subsequent research has supported Peirce in his claim that visual images and muscular imaginations provide the best thinking (NEM 4, 375; cf. Kent, 1997, 446).[2]

Creativity implies for Peirce the cooperation between abduction – the logical operation by which the first hypotheses are originated – and diagrammatic reasoning. An internal dialogue between images and ideas is needed because, without a verbal and logical description, our diagrams and hypotheses would be useless. Again, we can remember what Peirce wrote: creativity springs from an inner conversation not only with words but illustrated with diagrams and experiments like a lecture (CP 6.461, 1908).

In Peirce's scientific method, suitable for any research, different kinds of signs are combined in the abductive, deductive and inductive phases[3]; and it is necessary to assume that the deductive phase is as important as the abductive one. The experience is key to creativity, but so is the experimental analysis, which for Peirce plays a great part both in logic and in the history of science (CP 7.277, n.d.). If abduction allows contact with reality and provides us with the data from which the first ideas will emerge, deduction allows us to handle those ideas, to order them, analyze them and experiment on them without the limits of reality, so that all the fecundity they contain can be obtained. Knowledge cannot advance without abduction, but neither can we think in a rigorous and fertile manner without diagrammatic or deductive reasoning.

In conclusion, to reach the creative achievement, an imaginative play of ideas is required, for which the use of signs "of non-symbolic thought" is of great assistance: "pictures, diagrams or other images such as have to be used to explain the significations of words" (CP 6.338, 1909). For Peirce, truth is derived largely from the observation of creations of our own visual imagination, which we may set down in the form of diagrams (CP 2.77, c. 1902; 2.782, 1901; cf. Oostra, 2003, p. 8).

Some examples from Peirce's first trip to Europe

Consistent with his belief, diagrams and drawings abound in Peirce's writings and correspondence.[4] These are figures that clarify the meaning of the text or even form part of it. Peirce, in fact, stated that he thought in visual diagrams (Oostra, 2003, pp. 2–3; Kent, 1997).

Peirce made five trips to Europe – between 1870 and 1883 – as part of his scientific work at the service of the United States Coast and Geodetic Survey[5], and the letters from the first European tour in 1870 and 1871 are particularly rich in drawings. Those images are not something secondary to the thread of the narrative. They do not only illustrate what Peirce was describing, but also sometimes serve as diagrams that help to clarify thought and emphasize the ideas he wanted to convey. For example, in the letter from Rome on October 14th, 1870, to his mother, Peirce made a map of the route that he followed in the city – **Image 31**. That sketch not only served as an illustration of what he had just written, but also gave an idea of the length of the route he had done that day. With this drawing, Peirce reinforced the idea expressed in his words a few lines before: I "have been very successful today, I think, in utilizing every moment."

In several cases, Peirce's drawings were related to geometry or forms of the places he visited. On September 22nd, 1870, in a letter to his wife, Peirce drew the terraced vineyards in the hills of Sicily, whose horizontal lines produced a peculiar effect on the visitor. On page 4 of the same letter, Peirce made two drawings to illustrate his descriptions to his wife. In the first, he drew a diagram of the various craters of Etna, which were, in turn, small mountains, adding an exotic cactus below, “the Indian fig a tropical-looking juiceless thing.” In this last case and in other places, Peirce’s pictures became more figurative, trying to convey how something he had seen was. We could say they played the role of photographs, still uncommon in those days.

On the letter of September 2nd, 1870, Peirce drew for his mother a sophisticated form of a tablet covered with Arabic script in the mosque of Hagia Sophia – **Image 32**. Peirce wrote:

> In the mosque the tablets with Arabic writing on them excited the particular admiration of my friends & they declared that the art of Arabic chirography was on a level with painting & that such things were to be compared with the pictures of Raphael. There was one tablet which (...) I should have supposed to be a mere ornament, but they read it. (MS Am 1632, L 341)

But perhaps the most striking design and most unknown of the drawings included in those letters is the one he made on the back of his letter from Syracuse on September 22nd, 1870. Peirce drew with a pencil, as children sometimes like to do, the contour of his own right hand and pointed out numerous bites caused by fleas in the hotel where he was staying – **Image 33**. All this served to emphasize that “Syracuse is a disgusting place” and to beg for his wife’s compassion. The picture is much more graphic than any complaint and finely exemplifies the role of the drawings in his European letters.

Conclusion

Diagrams for Peirce are not only a way of expressing reasonings, but a way of making signs grow. Not only thought generates diagrams, but diagrams generate thought.

According to Peirce, thought is illustrated, and drawings and diagrams, built with the help of the imagination, are a central part of the activity of our mind. Images, beyond illustrating, compel us with a physical force. The diagrammatic representation allows us to order, understand, imagine and act upon the signs, that is, to continue thinking. In diagrams, therefore, resides the fertility of our reason and – along with abduction and induction – the creative achievement.

Notes

1 The notion of "reasonableness" in Peirce has been studied in: Barrena, S. (2007). La razón creativa. Crecimiento y finalidad del ser humano según C. S. Peirce, Rialp. The term "reasonableness" appears very late in Peirce's texts, since his usage of it covers only the ten years, between 1899 and 1908. Cf. Nubiola, J. (2009). What Reasonableness Really Is. *Transactions of the Charles S. Peirce Society* 45/2, 125–134.

2 In this sense, Peirce connects with the Medieval tradition of representing conceptual content in a schematic or visual way. Saint Bonaventure wrote: "Consequently perceptions received by the ears or by thought alone can be most easily retained if they are also conveyed by the mediation of the eyes, (...) so that we may keep hold by an act of sight, as it were, of things that we can scarcely embrace by an act of thought" (Bonaventure, 1891). People were expected not only to see the images, but to question them, play with them, and even subject them to a probationary investigation (Bouché, 328). Examples include the tree diagrams used by several thinkers, among others Raimundo Lull, which not only increased the meaning of philosophical questions but also represented intellectual operations and simulated natural phenomena, and could be seen as "diagrammatic experiments" (Higuera, 2019).

3 For a detailed explanation of Peirce's scientific method and its relation to creativity see: Barrena, S., & Nubiola, J. (2020). Abduction: The Logic of Creativity. In T. Jappy (Ed.), *The Bloomsbury Companion to Contemporary Peircean Semiotics* (pp. 185–203), Bloomsbury.

4 This topic has been developed by the authors in "Drawings, Diagrams, and Reasonableness. Peirce's Letters from his First Visit to Europe (1870–71)" (2012), in: F. Engel, M. Queisner, & T. Viola (Eds.), *Das bildnerische Denken: Charles S. Peirce* (pp. 175–185). Akademie Verlag.

5 These five trips to Europe and the European correspondence have been thoroughly studied by the Grupo de Estudios Peirceanos, Universidad de Navarra:
https://www.unav.es/gep/CorrespondenciaEuropeaCSP.html

9. Images of Knowledge from *Liber Calculationis* to Cerebro

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According to a philosophical tradition that can be traced back to Plato's *Theeetetus,* knowledge is justified true belief. Since beliefs are mental states, knowledge also is a sort of mental state. While literary narratives such as novels can effectively represent mental states of fictional characters, including their knowledge, visual narratives such as films are more apt to represent actions. Films deploy knowledge as a cue for representing actions rather than something that can be directly represented. Specifically, the quest for truth and knowledge is the source of action in films such as detective stories or legal dramas, whose protagonists aim to discover what the truth is about certain dramatic events. Documentaries are also often based on the quest for truth and knowledge, and so are fiction films such as *The Truman Show*, *The Village*, *Closer*, which involve lies and deceptions that the protagonists are meant to debunk.

The film genre that has knowledge at its core is science fiction. In his book *Metamorphoses of Science Fiction* (1979), Darko Suvin argues that science fiction involves not only fantastic entities but also pieces of knowledge that enable human characters to create those entities or at least to explain their existence. Science fiction films do not only exploit knowledge to trigger action as other genres do, they also aim to portray it. Specifically, in science fiction cinema, knowledge is embodied by characters such as the scientist or the engineer, places such as the laboratory or the spaceship, and a variety of high-tech objects. Among the latter, an especially interesting case is that of fictional artifacts used to visualize data in a way that mimics the functioning of real artifacts such as maps or models. Let me focus on a paradigmatic case. Cerebro is a fictional artifact in the X-Men comics and films – **Image 34**. This imaginary object can be compared with a historical artifact such as Liber Calculationis – **Image 35**, since both are used to visualize the location and calculate the movement of things in space and time. The key difference is that Liber Calculationis enables one to visualize the location and movement of inanimate objects, such as stars and planets, while Cerebro enables one to visualize the location and movement of living beings worldwide. Moreover, Cerebro allows one to distinguish human from the fantastic creatures that characterize the fictional world of X-Men, namely mutants.

Cerebro first appeared in the X-Men comic book #7, in 1964, and then was deployed in many other issues of the comic as well as in the films derived from them. In particular, the first two films of the X-Men series – *X-Men* (2000) and *X2* (2003), both directed by Bryan Singer – portray Cerebro as a laboratory that enables scientists to visualize a three-dimensional map whereby humans and mutants can be traced worldwide. As Jeffrey Kripal puts it in his book *Mutants and Mystics: Science Fiction, Superhero Comics, and the Paranormal* (2011, p. 208), Cerebro is "a futuristic superroom into which Professor Xavier wheels over a bridge in order to don the helmet that would magnify his already extraordinary telepathic powers and project the results onto the skull-like internal walls of the room."

Specifically, the laboratory is in a big, round room in the basement of Xavier's School for young mutants. A helmet enables Professor Charles Xavier to connect to Cerebro, thereby seeing images of the humans and the mutants whose brainwaves are captured by the device. While the images of humans are in black and white, those of mutants show up in red.

The main plot of the first film of the series, namely *X-Men*, revolves around Rogue, a girl who reveals an extraordinary mutant power that consists in absorbing the power and life force of others. Rogue is enrolled in Xavier's school but then leaves it. The power of Rogue is of extreme interest for Magneto, Xavier's colleague who created Cerebro with him but is now in conflict with him since Xavier struggles to achieve a deal between human and mutants while Magneto wants mutants to fight against humans. In a central scene of the film, Xavier uses Cerebro to find Rogue with the aim of preventing Magneto from capturing her. The eye control procedure opens the door of the spherical laboratory, and the mechanical voice of Cerebro welcomes Xavier, who explains to his ally Wolverine – and through him to the audience – how the machine operates: "Welcome to Cerebro. The brain waves of mutants are different from average humans. This device amplifies my power, allowing me to locate mutants across great distances. That's how I intend to find Rogue." Then Xavier wears the helmet and Wolverine leaves him alone in the room. The soundtrack mixes the "Cerebro theme" by composer Michael Kamen with the mechanical sounds of the machine mixes to emphasize the exceptionality of the moment. A mobile platform brings Xavier at the center of the room from which he can see the map of the humans and the mutants projected on the internal surface of the sphere. The climax of the scene is the appearance of Rogue (first in the map and then in the real place where she is), followed by the reverse shot of Xavier. The music then stops, and we see Xavier exiting the laboratory and telling his colleagues that he has found Rogue.

Later in the film, Xavier uses Cerebro again to locate Rogue, but a sabotage prevents him from projecting the map and makes him fall into a coma. Jean Grey, a pupil of Xavier who also has special mental powers, fixes Cerebro and uses it. The mix of Kamen's music and mechanical sounds in the soundtrack echoes the first scene in which we saw Xavier using the device, but in this case the machine is not shown operating. An ellipsis brings us to the moment in which the mutant Cyclops helps his partner Jean to recover after her first, upsetting Cerebro experience which has enabled her to locate Magneto.

The main antagonist of Xavier and his mutant allies in the sequel X2 is Military scientist Colonel William Stryker, who wants to build his own version of Cerebro with the aim of killing mutants. At the end of the scene in which Stryker says to Magneto "I'd like to have one final talk about the house that Xavier built and the machine called Cerebro," a cut leads us to Xavier entering Cerebro with Wolverine. This is the first time that Xavier allows Wolverine to remain in Cerebro while he is using it, and he explains to him in more detail how that works: "These lights represent every living person on the planet. White lights are humans. And these are the mutants. Through Cerebro, I'm connected to them. And they to me." Wolverine, in this scene, is again a proxy for the audience that can listen to a detailed explanation of the functioning of the machine which is portrayed by smooth camera movements emphasized by the solemn music composed by John Ottman, who is also the editor of the film. By using Cerebro, Xavier can locate Nightcrawler, a mutant with teleporting powers who attacked the President of the United States. Nightcrawler did so because he was brainwashed by Stryker's son, Jason, a mutant with mind-controlling powers

In the last act of X2, Stryker uses Jason to also brainwash Xavier and take control of Cerebro. Xavier enters Cerebro with Jason, who appears to him under the guise of a little girl. The editing alternates objective shots that portray Xavier and Jason inside the machine with subjectively inflected shots in which Xavier listens to the little girl he is hallucinating, who says to him: "Concentrate. Concentrate harder. Now, find them all. Find all the mutants. You, concentrate harder. Kill them. Kill them all." The turning point occurs when Magneto enters Cerebro and prevents Xavier from executing Jason's orders. Magneto would like to use Cerebro to exterminate humans instead of mutants, and he makes it so that Jason, under the guise of the little girl, says to Xavier: "There's been a change of plans. Now find them. Find them all – the humans. Find all the humans. Everyone [...] Find all the humans. Kill them" (Figure 1). Yet, the other mutants succeed in freeing Xavier from Jason's control, thereby foiling the massacre.

The centrality of Cerebro in X2 indicates that Cerebro is not just a device that mutants use but a fully-fledged protagonist of the X-Men saga on a par with mutants themselves. As its Latin name suggests, Cerebro is a gigantic brain that hyperbolically extends the human capacity of representing the world by means of maps and models. By being capable of tracing humans across the whole planet, Cerebro foreshadowed the Internet which did not exist in 1964, when Xavier's machine made its first appearance in the X-Men comics. At the beginning of the new century, when the first two X-Men films were released, the World Wide Web was already very popular but social networks had yet to be invented, hence Cerebro preserved its thoroughly fantastic nature. Nowadays, it rather looks like a fictional bridge between real historical artifacts such as Liber Calculationis and the pervasiveness of contemporary digital technologies that can locate us wherever we are.

10. Stock Images of Artificial Intelligence: Ontological, Ethical, and Aesthetical Implications

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In science communication, there is abundant use of stock images of AI, bought on the sites of agencies such as Getty Images and Shutterstock. In these images, one finds lots of clichéd and kitschy subjects about AI: humanoid robots touching computer screens, brains half made of electrical circuits, zeros and ones falling from the sky, and of course hundreds of variations of Michelangelo's *The Creation of Adam* in a human-robot version. Most of the criticism of these images focuses precisely on their subjects. But what if the real ethical problem was not in the subjects but rather in the background? What if a major ethical issue was, for instance, the abundant use of the color blue in the background of these images? This is the thesis we would like to discuss in detail in this article.

Our research has been focused on the topic of AI images for some time now. In particular, what interests us are the stock images of AI, that is, those that can be bought on sites like Getty Images and Shutterstock. Stock images are usually ignored by researchers because they are considered the "wallpaper" of our consumer culture. Yet, they are everywhere. Stock images of emerging technologies such as AI (but also quantum computing, cloud computing, blockchain, etc.) are widely used, for example, in science communication and marketing contexts: conference announcements, book covers, advertisements for university masters, etc. There are at least three reasons for me to take these images seriously.

The first reason is "ontological." The philosophy of technology (the discipline one of us is concerned with) has been characterized in the last 30 years by what is called its "empirical turn" (Brey, 2010)[1]. This name refers to how a whole new generation of philosophers of technology between the 1980s and 1990s began to criticize the way in which older generations talked about technology. Authors such as Martin Heidegger in the 1950s criticized modern technology without knowing anything about it, having never visited a factory or a research laboratory, or discussed the technology with the engineers who built it. Famous in this regard is Heidegger's statement that "the essence of technology is by no means anything technological." But are things really like that? Is it really possible to talk about (and criticize) technology without knowing how it works, without at least asking the experts? The new philosophers of the "empirical turn" began to be interested "in the things themselves," that is, in concrete technologies. In contrast with the "macro" perspective of the previous generation, they began to prefer a "micro" perspective. Yet, our thesis is that they ended up throwing the baby out with the bathwater. By focusing on what is visible "up close," they also foreclosed themselves from looking at things "from afar." Our idea, of course, is not to go back to the old philosophy of technology, but to articulate a focus on the micro with a focus on the "conditions of possibility" in which the "micro" of technology (i.e., technology as a mere artifact) already finds itself (Romele, 2021). These conditions of possibility can be of at least three orders: techno-scientific, social-economic, and linguistic-cultural.

As for the first order, it is almost trivial to say that my smart phone would not be what it is, indeed would not even exist, if there were no elements such as the following: other phones to call, satellites, undersea cables, antennas, information theory, behavioral studies, design, ergonomics, etc. As for the second order, it is just a matter of making the list longer: contracts with phone companies, Google, Apple, and, above all, us, as users (of technology), consumers (of services), and producers (of data). As for the third order, we would say that every technology is linked to a "cultural atmosphere": this means, first of all, that it is absorbed by a culture. Think of how the use of the smart phone has adapted over time to national habits – we are thinking of the Italians who make "noisy" use of the *telefonino*, as they call it. This also means that being absorbed by a culture, technologies like the smart phone also transform it. No culture is immune to the technologies it appropriates. Finally, this means that in order to cope with specific technologies, a culture produces representations of them. These representations in turn are not simply copies of technological reality but are also, and above all, crystallizations of the expectations and imaginaries that a culture has about these technologies.

Now, it is precisely the linguistic-cultural order and, in this, the theme of representations, expectations and imaginaries that interests me. Our "ontological" idea with respect to AI is that when one wants to define AI, one cannot but include this level – just as one cannot exclude the other "conditions of possibility." AI is not only a technological fact but also a cultural fact. Cultural representations of AI are not just phantasmagoria detached from technological reality but play an active role in the processes of technological innovation.

This leads us to explain the second reason for our interest in the stock images of AI, which is "ethical-political" (Romele, forthcoming). It is interesting to note that even the most careful AI ethicists pay little attention to the way AI is represented and communicated, both in scientific and popular contexts. For instance, a volume of more than 800 pages like the *Oxford Handbook of Ethics of AI* does not contain any chapter dedicated to the representation and communication, textual or visual, of AI; however, the volume's cover image is taken from iStock, a company owned by Getty Images. The subject of it is a classic androgynous face made of "digital particles" that become a printed circuit board. The most interesting thing about the image, however, is not its subject (or figure, as we say in art history) but its background. We take this focus on the background rather than the figure from the French philosopher Georges Didi-Huberman (2005) and, in particular, from his analysis of Fra Angelico's painting – **Image 36**.

Didi-Huberman devotes some admirable pages to Fra Angelico's use of white in his fresco of the Annunciation painted in 1440 in the convent of San Marco in Florence. This white, present between the Madonna and the Archangel Gabriel, spreads not only throughout the entire painting but also throughout the cell in which the fresco was painted. Didi-Huberman's thesis is that this white is not a lack, that is, an absence of color and detail. It is rather the presence of something that, by essence, cannot be given as a pure presence, but only as a "trace" or "symptom." This thing is none other than the mystery of the Incarnation. Fra Angelico's whiteness is not to be understood as something that invites absence of thought. It is rather a sign that "gives rise to thought,"[2] just as the Annunciation was understood in scholastic philosophy not as a unique and incomprehensible event, but as a flowering of meanings, memories, and prophecies that concern everything from the creation of Adam to the end of time, from the simple form of the letter M (Mary's initial) to the prodigious construction of the heavenly hierarchies.

Image 37 collects about 7,500 images resulting from a search for "Artificial Intelligence" in Shutterstock. It is an interesting image because, with its "distant viewing," it allows the background to emerge on the figure. In particular, the color of the background emerges. Two colors seem to dominate these images: white and blue. Our thesis is that these two colors have a diametrically opposed effect to Fra Angelico's white. If Fra Angelico's white is something that "gives rise to thought," the white and blue in the stock images of AI have the opposite effect.

Consider the history of blue as told by French historian Michel Pastoureau (2001). He distinguishes between several phases of this history: a first phase, up to the 12^{th} century, in which the color was almost completely absent; an explosion of blue between the 12^{th} and 13^{th} centuries (consider the stained glass windows of many Gothic cathedrals); a moral and noble phase of blue (in which it became the color of the dress of Mary and the kings of France); and finally, a popularization of blue, starting with Young Werther and Madame Bovary and ending with the Levi's blue jeans industry and the company IBM, which is referred to as the Big Blue. To this day, blue is the statistically preferred color in the world. According to Pastoureau, the success of blue is not the expression of some impulse, as could be the case with red. Instead, one gets the impression that blue is loved because it is peaceful, calming, and anesthetizing. It is no coincidence that blue is the color used by supranational institutions such as UN, UNESCO, and European Community, as well as Facebook and Meta, of course. In Italy, the police force is blue, which is why policemen are disdainfully called "Smurfs" – **Image 38**.

If all this is true, then the problem with stock AI images is that, instead of provoking debate and "disagreement," they lead the viewer into forms of acceptance and resignation. Rather than equating experts and non-experts, encouraging the latter to influence innovation processes with their opinions, they are "screen images" – following the etymology of the word "screen," which means "to cover, cut, and separate." The notion of "disagreement" or "dissensus" (*mésentente* in French) is taken from another French philosopher, Jacques Rancière (2004), according to whom disagreement is much more radical than simple "misunderstanding (*malentendu*)" or "lack of knowledge (*méconnaissance*)." These, as the words themselves indicate, are just failures of mutual understanding and knowledge that, if treated in the right way, can be overcome. Interestingly, much of the literature interprets science communication precisely as a way to overcome misunderstanding and lack of knowledge. Instead, we propose an agonistic model of science communication and, in particular, of the use of images in science communication. This means that these images should not calm down, but rather promote the flourishing of an agonistic conflict (i.e., a conflict that acknowledges the validity of the opposing positions but does not want to find a definitive and peaceful solution to the conflict itself)[3]. The ethical-political problem with AI stock images, whether they are used in science communication contexts or popular contexts, is then not the fact that they do not represent the technologies themselves. If anything, the problem is that while they focus on expectations and imaginaries, they do not promote individual or collective imaginative variations, but rather calm and anesthetize them.

This brings us to our third reason for talking about stock images of AI, which is "aesthetic" in nature. The term "aesthetics" should be understood here in an etymological sense. Sure, it is a given that these images, depicting half-flesh, half-circuit brains, variants of Michelangelo's *The Creation of Adam* in human-robot version, etc., are aesthetically ugly and kitschy. But here we want to talk about aesthetics as a "theory of perception" – as suggested by the Greek word *aisthesis*, which means "perception", precisely. In fact, we think there is a big problem with perception today, particularly visual perception, related to AI. In short, we mean that AI is objectively difficult to depict and hence make visible. This explains, in our opinion, the proliferation of stock images.

We think there are three possible ways to depict AI (which is mostly synonymous with machine learning) today: (1) the first is by means of the algorithm, which in turn can be embedded in different forms, such as computer code or a decision tree. However, this is an unsatisfactory solution. First, because it is not understandable to non-experts. Second, because representing the algorithm does not mean representing AI: it would be like saying that representing the brain means representing intelligence; (2) the second way is by means of the technologies in which AI is embedded: drones, autonomous vehicles, humanoid robots, etc. But representing the technology is not, of course, representing AI: nothing actually tells us that this technology is really AI-driven and not just an empty box; (3) finally, the third way consists of giving up representing the "thing itself" and devoting ourselves instead to expectations, or imaginaries. This is where I would put most of the stock images and other popular representations of AI.[4]

Now, there is a tendency among researchers to judge (ontologically, ethically, and aesthetically) images of AI (and of technologies in general) according to whether they represent the "thing itself" or not. Hence, there is a tendency to prefer (1) to (2) and (2) to (3). An image is all the more "true," "good," and "aesthetically appreciable" the closer it is (and therefore the faithful it is) to the thing it is meant to represent. This is what I call "referentialist bias." But referentialism, precisely because of what we said above, works poorly in the case of AI images, because none of these images can really come close to and be faithful to AI. Our idea is not to condemn all AI images, but rather to save them, precisely by giving up referentialism. If there is an aesthetics (which, of course, is also an ethics and ontology) of AI images, its goal is not to depict the technology itself, namely AI. If anything, it is to "give rise to thought," through depiction, about the "conditions of possibility" of AI, i.e., its techno-scientific, social-economic, and linguistic-cultural implications.

We want to conclude this already too-long text with a methodological note. We said at the beginning that our challenge is not to return to the old philosophy of technology (the one that criticizes technology without knowing anything about it), but to articulate the "micro" and "macro" perspectives. This is precisely what we try to do with AI stock images. In fact, alongside theoretical work such as the one we discuss above, we also try to conduct empirical research on these images. We showed earlier an image that is the result of work we have recently conducted. In this work, we first used the web crawler Shutterscrape, which allowed us to download massive numbers of images and videos from Shutterstock. We obtained about 7,500 stock images for the "Artificial Intelligence" search. Second, we used PixPlot, a tool developed by Yale's DH Lab. The result is accessible through the link in the footnote.[5] The map is navigable: you can select one of the ten clusters created by the algorithm and, for each of them, you can zoom and de-zoom, and choose single images. We also manually labeled the clusters with the following names: (1) background, (2) robots, (3) brains, (4) faces and profiles, (5) labs and cities, (6) line art, (7) Illustrator, (8) people, (9) fragments, and (10) diagrams.

What seems interesting to us in this experimentation is the fact of leaving an AI algorithm (such as the one at the base of PixPlot) to the task of "understanding" its own representations. Another empirical work, which we did with other colleagues (Marta Severo, Olivier Buisson and Claude Mussou), goes in the opposite direction. In this case, we used a tool called Snoop,[6] developed by INA and INRIA and also based on an AI algorithm. Behind Snoop, there is the idea that the researcher, like a "turker" but aware of the goals of their work, can train the algorithm to recognize classes of images within a corpus. While with PixPlot the choice of the clusters is automatic, with Snoop the classes are decided by the researcher and the class members are found by the algorithm. Snoop allowed us, for example, to distinguish, in the class of white robots, the two subclasses of white female robots and white infantile robots.

Finally, there's another little project of which we are particularly fond. It is the Instagram profile ugly.ai.[7] Inspired by existing initiatives such as the NotMyRobot! Twitter profile, ugly.ai wants to monitor the use of AI stock images in science communication and marketing contexts. The project also aims to raise awareness among both stakeholders and the public of the problems related to the depiction of AI (and other emerging technologies) and the use of stock imagery in particular. What we would like to say in conclusion is that all these empirical experiments are not external to our theoretical reflection. In fact, we are convinced that it is precisely these empirical works, these "explanations," that lead us each time to "better understanding" the transcendental implications of depicting AI.

Notes

1 That a philosopher of technology speaks of images of technology is in itself interesting. In fact, images of technology (not the images produced by scientific instruments – of which there are a great deal – but rather the images of technology itself) are mostly relegated to the role of idols, that is, of deceptive or at least dangerous images, because they are copies of those very things to which empirical philosophers of technology want to devote themselves. The job of looking at images of technology is then left, with grave consequences, to other disciplines.

2 The expression is borrowed from Ricoeur (1967).

3 On the agonistic model, inspired by Chantal Mouffe's philosophy, in science and technology, see Popa, Blok, and Wessenlink (2020).

4 Needless to say, this is an idealistic distinction, in the sense that these levels are mostly overlapping: algorithm codes are colored, drones fly over green fields and blue skies that suggest hope and a future for humanity, and stock images often refer, albeit vaguely, to existing technologies (touch screens, networks of neurons, etc.).

5 https://rodighiero.github.io/AI-Imaginary/#.

6 https://hal.archives-ouvertes.fr/hal-02096036.

7 https://www.instagram.com/ugly.ai/.

11. Are We All Narcissists? The Pseudo-Narcissism of the Internet

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Journalists and psychologists claim that we are more narcissistic than ever, especially when we started to make extensive use of social networks and digital tools. They are probably wrong: social networks and other digital tools don't increase our self-love, but amplify our quest for the approval of others. René Girard's theory contradicts psychoanalysis Sigmund Freud by stating that digital users are not narcissistic but pseudo-narcissistic, opening the new era of pseudo-narcissism.

The term "narcissism" derives from a Greek legend whose most notable version is told by the Roman poet Ovid in the Metamorphoses. In this legend, Narcissus is a man of exceptional beauty, so immensely proud of his appearance that he refuses any female or male suitors. One day, Nemesis, the goddess who enacts retribution against those who succumb to arrogance, decides to listen to the many victims of rejection and avenge them: she forces Narcissus to be infatuated with his reflection in the waters of a pond. He contemplates his image for days, desperate to catch his reflection, until he finally dies in the waters.

Today the American population is experiencing an increase of narcissistic behaviors, recent studies conducted by experimental psychologists say. The topic, which is a subject of great debate not only in academia but also in popular magazines, concerns Internet users and more specifically those who make use of social networks. A few years ago, in The New York Times, professor Jean Twenge from San Diego University shed light on a contemporary disease that affects the Internet, claiming that the intensive use of social networks is provoked by a pathological narcissism.

However, an attentive reader might already have noticed that the story is missing a second point of view, Echo's one. The nymph Echo was one of Narcissus' female suitors – **Image 39**. When Narcissus died in the waters under Echo's very eyes, she kept repeating "Alas! Alas!" Her standpoint is useful to introduce the concept of pseudo-narcissism by René Girard. Girard is a philosopher of social sciences who coined theories applicable to different domains; he was admired by the co-founder of PayPal Peter Thiel as well as by his colleague Michel Serres, who dubbed him the Darwin of social sciences. When Girard was at John Hopkins University, he developed the theory of mimetic desire, which states that "man is the creature who does not know what to desire, and he turns to others in order to make up his mind. We desire what others desire because we imitate their desires" (Girard, 1987, p. 122). Recontextualizing this statement in Narcissus' story, it can be said that Narcissus is actually a pseudo-Narcissus because he is not attracted by his image but rather by the desire of the suitors who foster his own love. Narcissus, indeed, is attracted by Echo's desire.

Narcissists are only apparently self-sufficient, voluntarily masking the imitation of others' desires. On Instagram, influencers are not in love with themselves but with their image in the eyes of their followers. "The subject desires himself by imitating others who desire him," states Jean-Pierre Dupuy (1989, p. 494). The followers are contaminated by the pseudo-narcissist's self-love, which they imitate as positive feedback by reinforcing the apparent narcissism of the pseudo-narcissist. Or influencers imitate the followers' desire by taking themselves as a reference and expressing indifference toward others through a fake manifestation of desire. Self-lovers constantly need to reestablish the attraction toward themselves by provoking it.

Despite the suitability of pseudo-narcissists to social media, the Freudian theory of narcissism at its odds is still the most acclaimed on the Internet. Based on the Greek myth of Narcissus, the founder of psychoanalysis Sigmund Freud built a theory of narcissism stating that "the individual acts as if he were in love with himself" (Freud, 1919, Ch. 3.3). According to Freud, narcissism in adults is a pathology that entails renouncing the outside world and redirecting desire towards the individual, also known in the Freudian terminology as the ego. The investment of desire in oneself implies that the narcissist doesn't require approval, exerting only a great charm on others. The more one loves oneself, the more one is supposed to be popular and praised.

René Girard harshly criticized the Freudian analysis of narcissistic individuals (1965, pp. 105–107), stating that Freud doesn't explain why narcissists attract non-narcissistic individuals (Dupuy, 2005). The theory of mimetic desire offers a simple solution: narcissists don't direct a self-generated desire on themselves but rather imitate the desire of others toward their person. Narcissists, indeed, do not admire themselves *ex nihilo*. Narcissus doesn't attract Echo from the start, but he falls in love with Echo's desire for him, becoming self-absorbed as if he had always been – **Image 40**.

Today we have more technology than ever in history to explore our bodies and activities, but are we really self-admiring? According to Girard, we admire our reflections in others' digital eyes. When we share posts on social networks, we are eager to get rewarded by shares and likes to compare numbers of followers between accounts, to compete for viewers on YouTube, and to break records on Strava and other sports apps. Are we not worried about becoming popular? Have we not already entered the competition for popularity?

The digital era amplified the formation not of narcissists, but of pseudo-narcissists craving visibility and in competition with each other on the worldwide scale of the Internet. We are more than ever in need of the remote others.

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